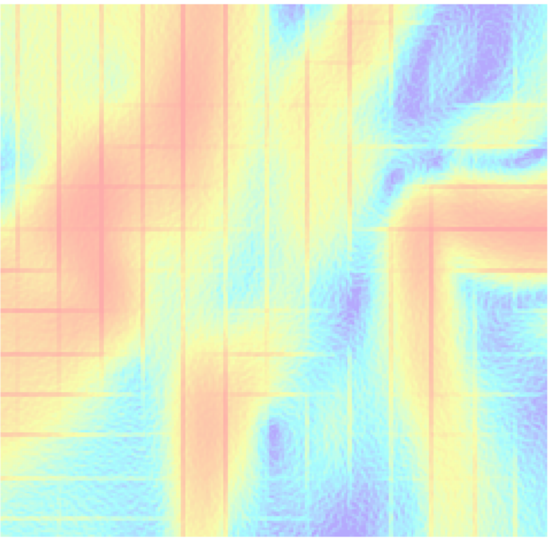
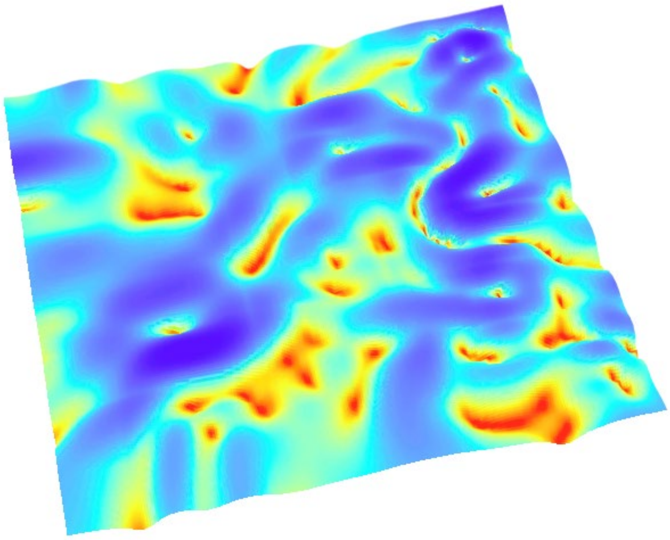
THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

DEPARTMENT OF LAND SURVEYING AND GEO-INFORMATICS
土地測量及地理資訊學系

LSGI3242A Digital Terrain Modelling





LAB 8 – EFFECT OF DEM ACCURACY ON SLOPE VALUES

Report

Abstract

Slope is one of the major components on the terrain surface as it indicates the change of elevation in a location. In the field of digital terrain modeling, analysis can be performed using a raster DEM to show the slope pattern of a certain region. This report will demonstrate slope calculation and the effect of random errors on slope value.

IP Pui Ling 18050032D

Content

1. Introduction

In the field of digital terrain modelling, a terrain surface can be constructed by using sets of sample data. The terrain surface can be represented in raster format and can be used for further analysis. One of the major tasks in digital terrain analysis is the calculation of slope and investigating effect of random errors on slope value.

ArcGIS Pro provides a sets of surface tools to allow users to quantify and visualize a terrain landform represented by a digital elevation model. It identifies the steepness at each cell of a raster surface. Such calculation of slope can facilitate assessment in environmental and construction aspect.

In this practice, the use of slope tool will be demonstrated to investigate the steepness of the terrain concerned. Then, comparison on different slope generation methods, effect of errors of DEM on slope calculation and comparison on color schemes in gradient indication will be explored in the result analysis part. Lastly, ideas will be raised regarding the employment of slope value to conduct further hydrology analysis.

2. Related Concept

2.1. Slope Calculation Unit and Method

In the 'Slope' analysis tool, ArcGIS Pro provides different ways for generating slope. Slope can be generated by two types of units, degrees or percentage, and by two methods, planar or geodesic.

2.1.1. Calculation Unit – Degree and Percentage

The generation unit determines the measurement units of the output slope raster. There are two types of units available – namely degree and percentage.

- Degree is to calculate the inclination of slope in degree
- Percentage is to calculate the slope in percent rise – dividing the rise by the run and then multiplying the result by 100. When the percentage is 100%, the rise equals to the run.

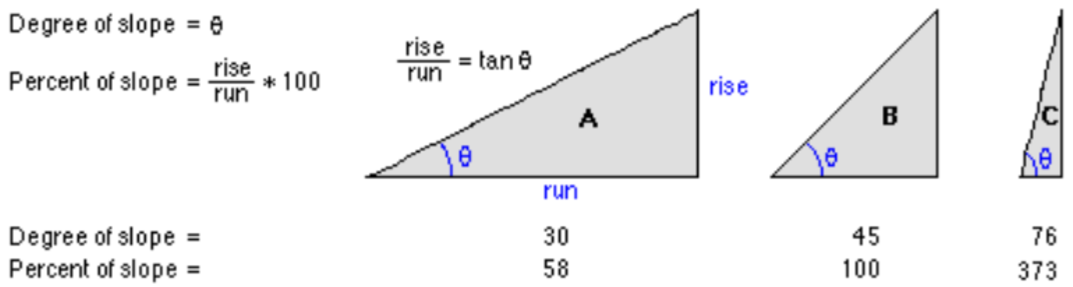


Figure 1. Slope value calculated using degree and percentage unit.

2.1.2. Calculation Method – Planar and Geodesic

The generation method determines whether to calculate the aspect based on a planar or geodesic method.

- ✧ Planar calculates the slope on a projected flat plane using a 2D Cartesian coordinate system. The slope value is calculated using the average maximum technique. For each cell, the tool calculates the maximum rate of change in value from that cell to its neighbors. Basically, the maximum change in elevation over the distance between the cell and its eight neighbors identifies the steepest downhill descent from the cell.

- ✧ Geodesic calculates the slope in geocentric 3D coordinate system, as known as the Earth Centered, Earth Fixed (ECEF) coordinate system, by considering the shape of earth as an ellipsoid.

  The geodesic slope is the angle formed between the topographic surface and ellipsoid surface. Any surface parallel to the ellipsoid surface has a slope of 0. To calculate the slope at each location, a 3 by 3 cell neighborhood plane is fitted around each processing cell using the Least Squares Method (LSM).

  Geodesic method will use the z-unit of the input raster if they are defined in the spatial reference. The geodesic method produces a more accurate slope than the planar method.

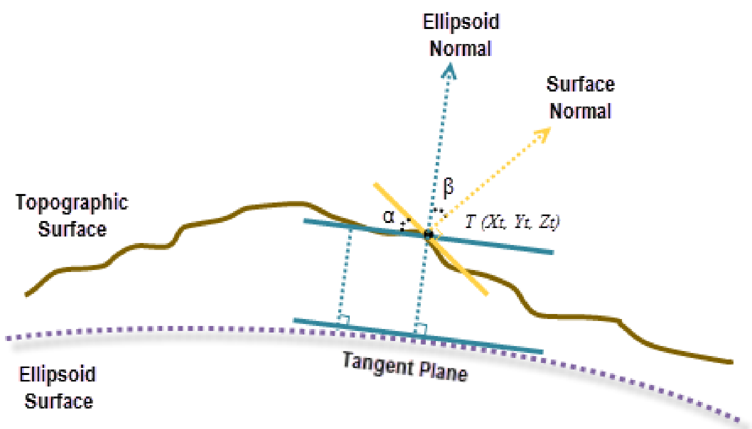


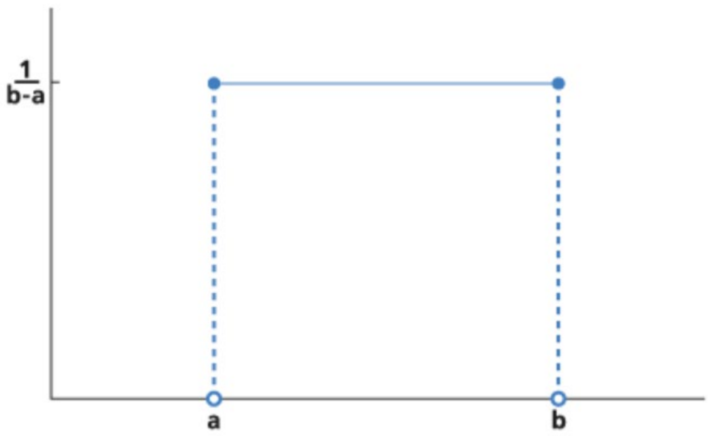
Figure 2. The height for calculation is the ellipsoid height referenced to the ellipsoid surface.

2.2. Distribution Methods of Random Raster

When generating random raster, the values assigned to each cell in the output raster are derived from rando number generator and the selected distribution type. The values of the cell are randomly determined and the values fall between 0 and 1, and each value is independent from other cell values. Various distribution types are available for random raster generator when assigning value in the output raster and they can produce different results.

2.2.1. Uniform Distribution

Uniform distribution is a continuous probability distribution that all values within a specified interval have the same probability. It is often used to model random events when each potential outcome or occurrence has equal probability of occurring. The random values selected are between the minimum and maximum (exclusive). If no minimum and maximum are provided, uniform variables between 0.0. and 1.0 are produced.



$$f(x) = \begin{cases} \frac{1}{b-a} & for(a \leq k \leq b) \\ 0 & for(x < a)or(x > b) \end{cases}$$

a = minimum value

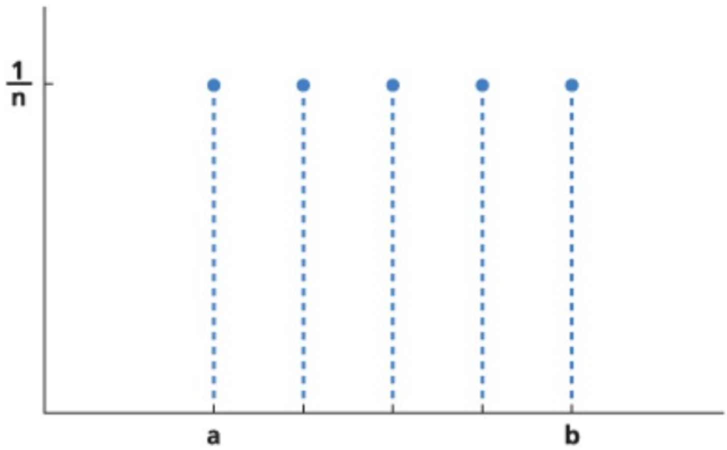
b = maximum value

for interval with equal probability

Figure 1. Uniform distribution (ArcGIS Pro Help).

2.2.2. Integer Distribution

Integer distribution is the probability distribution that all discrete values within a specified interval have the same probability. It can be said as the discrete version of uniform distribution. Integer distribution is often used to model random events when each potential outcome or occurrence has equal probability of occurring. The random values selected are between the minimum and maximum. If no minimum and maximum (exclusive) are provided, uniform values between 1 to 100 are produced.



$$f(x)=\begin{cases}\frac{1}{b-a} & for(a\le x\le b)\\ 0 & otherwise\end{cases}$$

a = minimum value

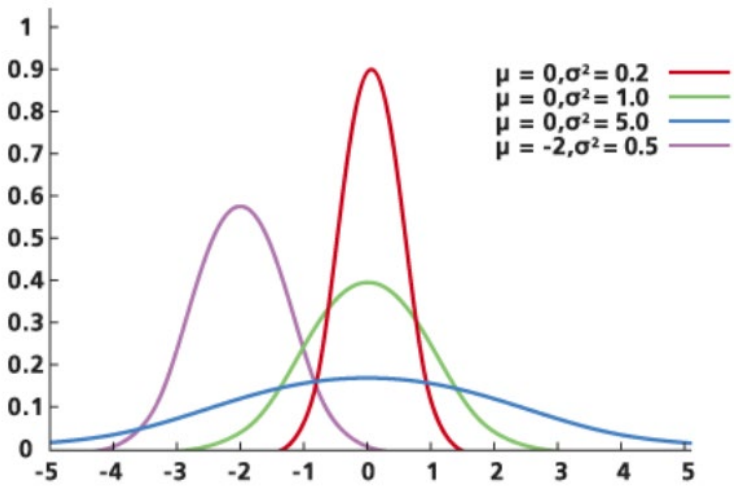
b = maximum value

for interval with equal probability

Figure 2. Integer distribution (ArcGIS Pro Help).

2.2.3. Normal Distribution

Normal distribution models continuous random variables that commonly occur. It is built on the central limit theorem, which is based on the principle that the sum of the random variables is normally distributed if there are a large number of observations.



$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

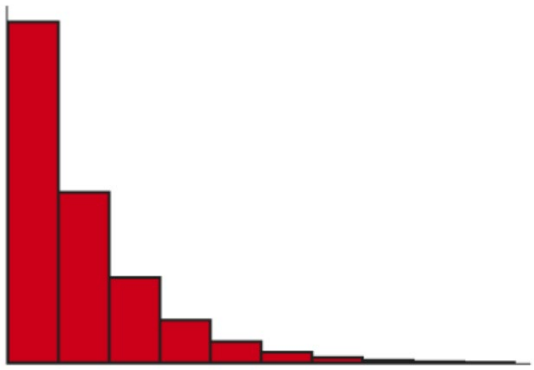
μ = mean

σ = standard deviation (+ve)

Figure 3. Normal distribution (ArcGIS Pro Help).

2.2.4. Geometric Distribution

Geometric distribution is a discrete probability distribution. There are two major types of phenomena that it models – (1) the probability of the number of times it takes for a success, and (2) the probability of the number of failures before a success. The probability of the occurrence of events is independent of one another.



$$P(X = n) = (1 - p)^{n-1}\, p$$

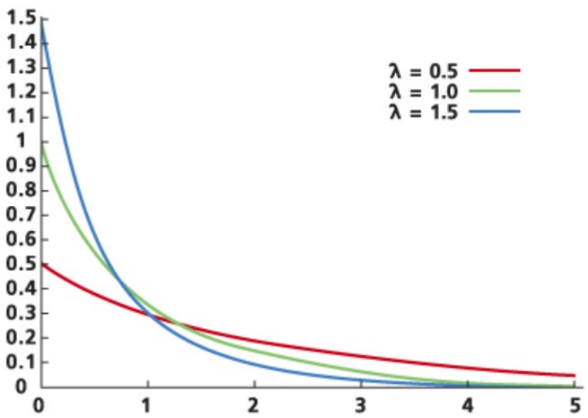
p = probability of success

n = number of trials

Figure 4. Geometric distribution (ArcGIS Pro Help).

2.2.5. Exponential Distribution

Exponential distribution is a continuous probability distribution. It is generally used to model time between events that occur at a constant coverage rate, or the distribution can be used to model occurrence of events in a per-unit distance. As the time or distance increases, there is an exponentially greater chance for the state to change or the event to occur. The occurrences of the events are independent to one another.



$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

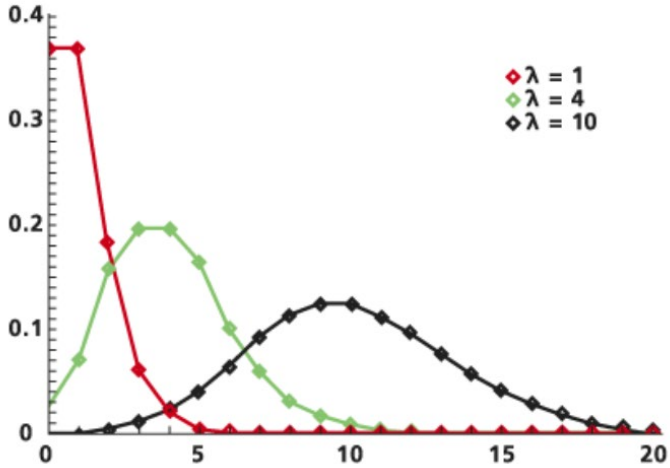
e = natural logarithm

x = possible number of occurrences for the event (+ve)

Figure 5. Exponential distribution (ArcGIS Pro Help).

2.2.6. *Poisson Distribution*

Poisson distribution is a discrete probability distribution modeling the probability of the number events occurring over a fixed time step given a known mean. The events are independent from the last time they occurred. The distribution is sometimes called the law of small number because the event does not happen often, but there are many opportunities for it to occur.



$$f(x;\lambda)=\begin{cases}\lambda e^{-\lambda x}, & x\geq 0\\ 0, & x<0\end{cases}$$

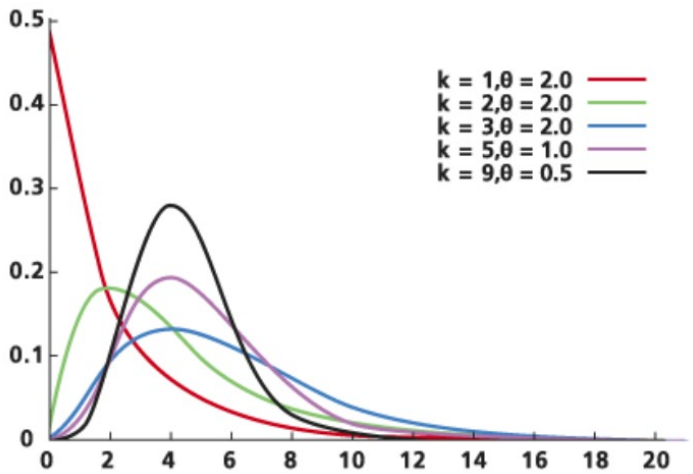
e = natural logarithm

k = possible number of occurrences for the event (+ve)

Figure 6. Poisson distribution (ArcGIS Pro Help).

2.2.7. *Gamma Distribution*

Gamma distribution is a continuous probability distribution modeling the sum of multiple independent, exponentially distributed variables. It can be viewed as a special case of exponential distribution.

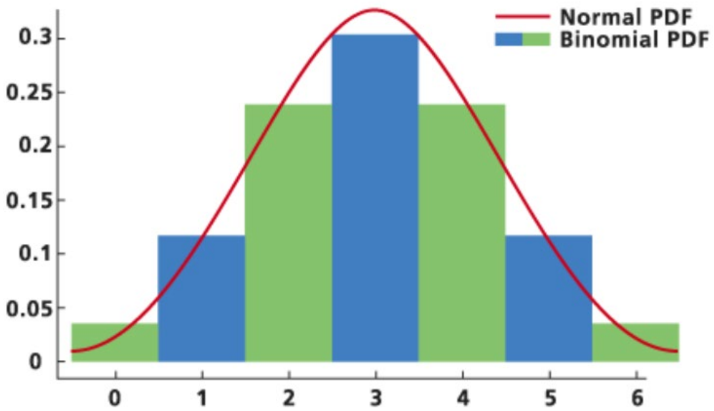


$$f(k;\lambda)=x^{k-1}\frac{\exp\left(-x/\theta\right)}{\Gamma(k)\theta^{k}}$$

Figure 7. Gamma distribution (ArcGIS Pro Help).

2.2.8. Binomial Distribution

Binomial distribution models the number of occurrences of an event when observing a sequence of potential producers of the event. It describes occurrence but not magnitude. Binomial distribution returns a random variable for the number of successes out of n trials where the probability for success in each trial is p.



$$f(x) = \frac{n!}{x!(n-x!)} p^x (1-p)^{n-x}$$

n = number of observations

p = probability of occurrence

x = number of successes ranging from 0 to n

Figure 8. Binomial distribution (ArcGIS Pro Help).

2.2.9. *Negative Binomial Distribution*

Negative binomial distribution is a discrete probability distribution based on Bernoulli trials which model events in which trials have one of two outcomes (success and failure). They are independent from one another.

$$f(k) = \frac{\Gamma(r+k)}{k!\Gamma(r)} p^r (1-p)^k$$

r = number of failures

p = probability of success

k = number of success ranging from 0 to n

3. Methodology

The two sample maps from the previous lab are used in this practice.

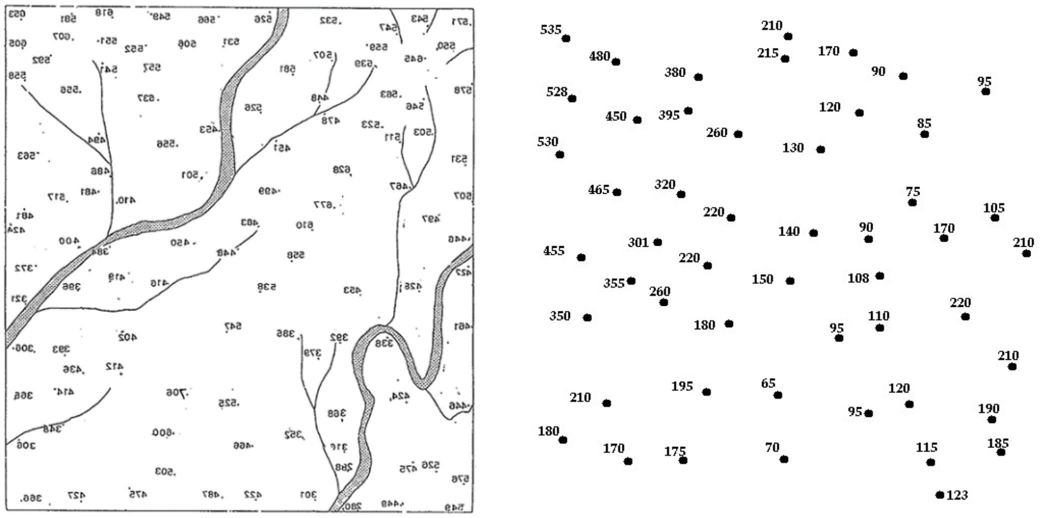
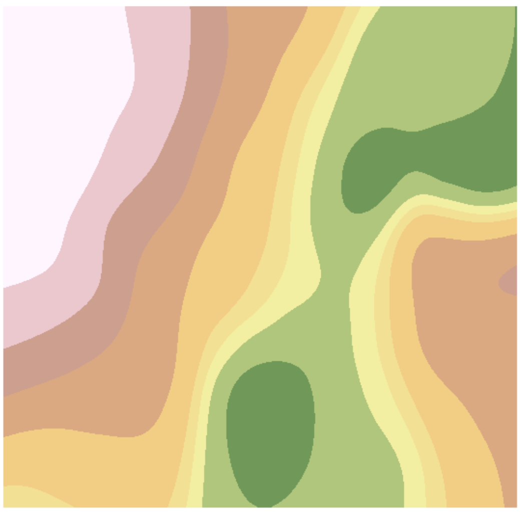


Figure 9-10. Two sample maps used in this practice.

In Lab 6, DEMs are interpolated by IDW, kriging and spline. In this practice, the raster surface created by spline is used for performing slope calculation as spline has the best performance among the three interpolation approaches in DEM generation.

Sample 1

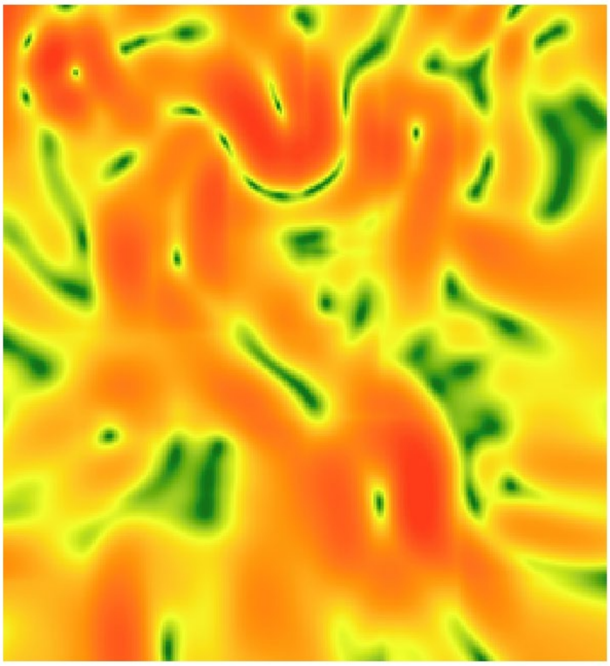


Sample 2

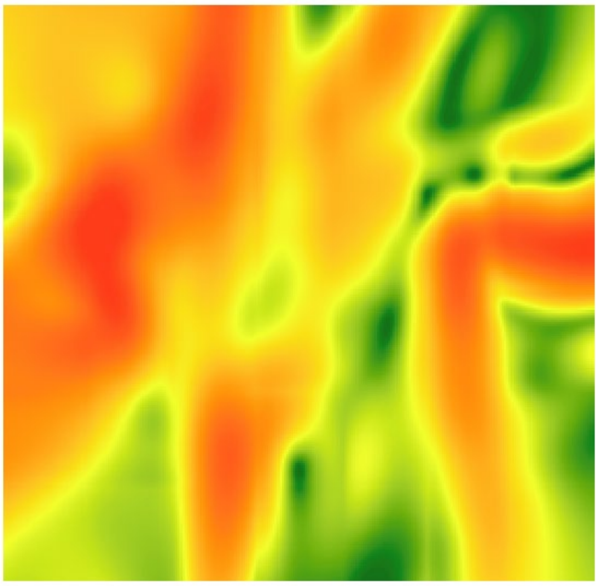
3.1. Calculation of Slope

Slope calculation can be done with the 'Slope' tool chosen from 'Geoprocessing' panel. It takes a DEM surface as input and returns a raster showing the steepness of slope of each cell.

The resulted slope maps:



Sample 1



Sample 2

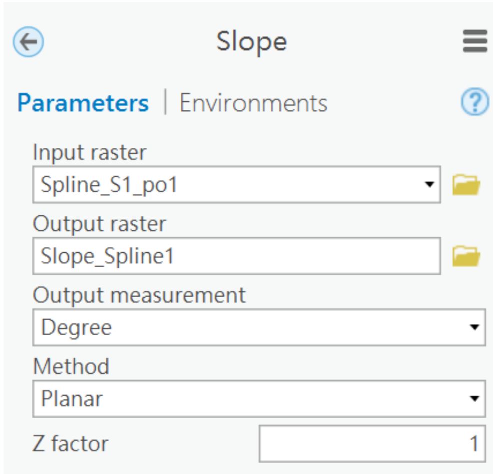


Figure 11. Input a raster surface in 'Slope' panel for slope calculation.

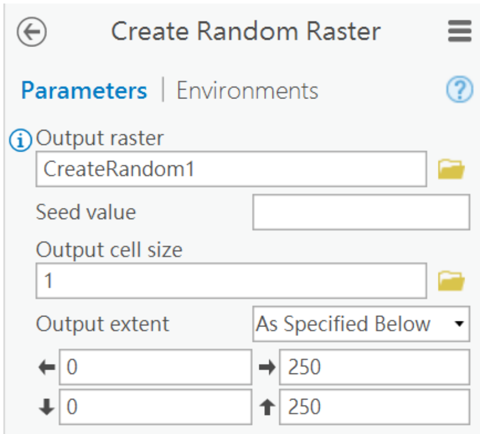
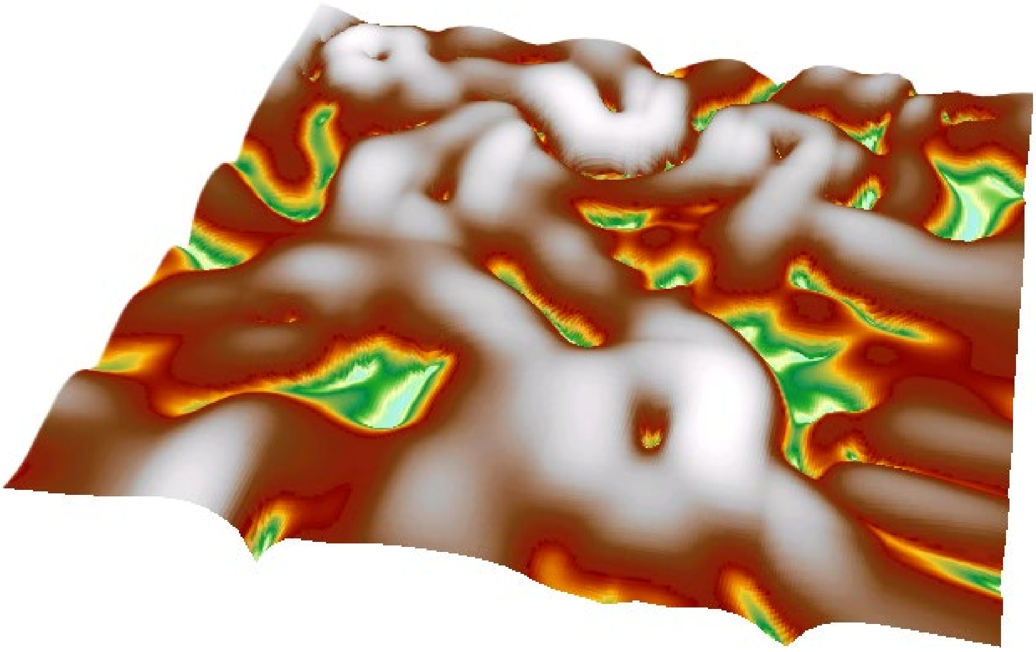
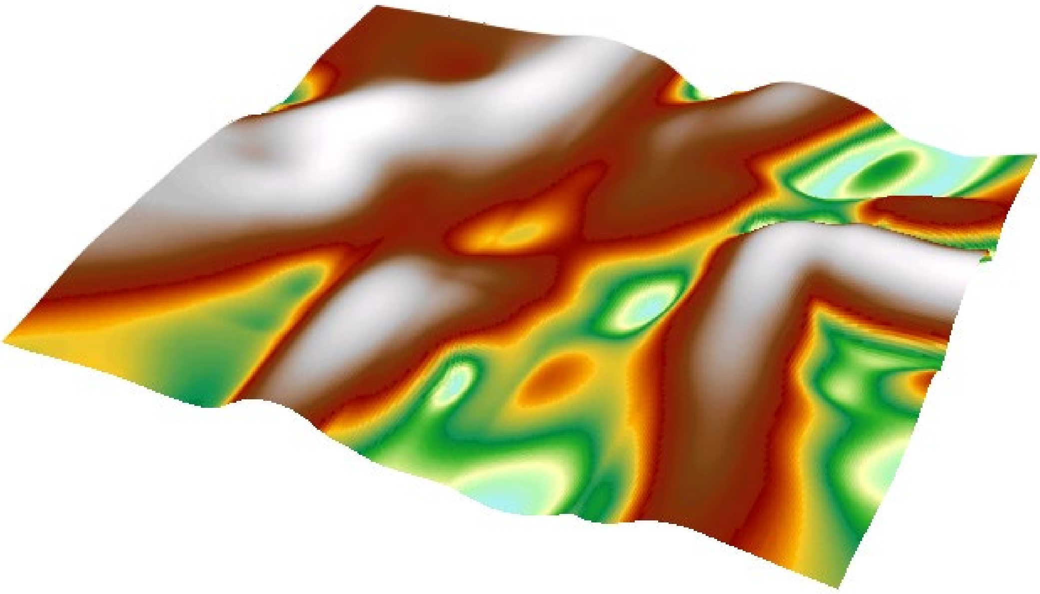


Figure 12. 'Create Random Raster'

The resulted slope maps of 3D view:



Sample 1

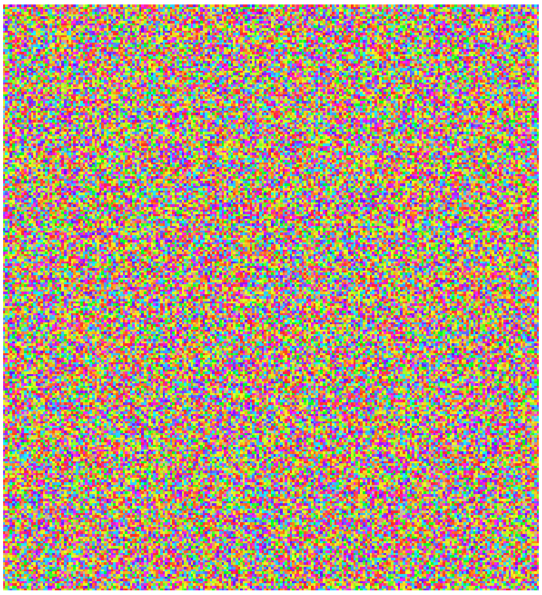


Sample 2

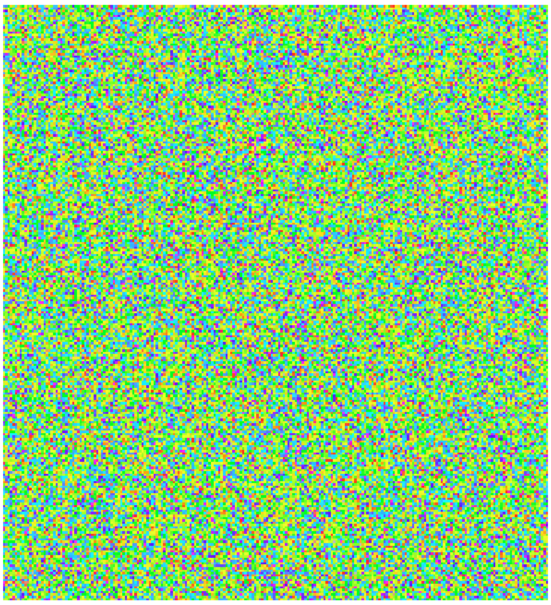
3.2. Creation of Random Noise

To investigate how random noise affects the calculation of slope, random raster has to be generated by 'Create Random Raster' tool (Figure. 12) with difference distribution types first.

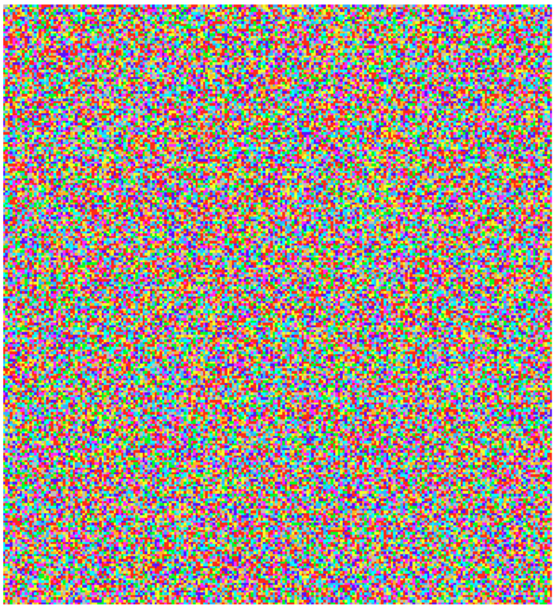
The random raster created:



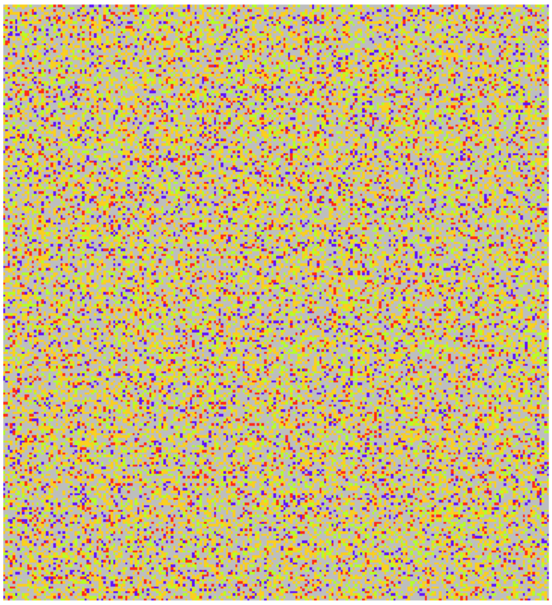
Uniform



Normal



Integer

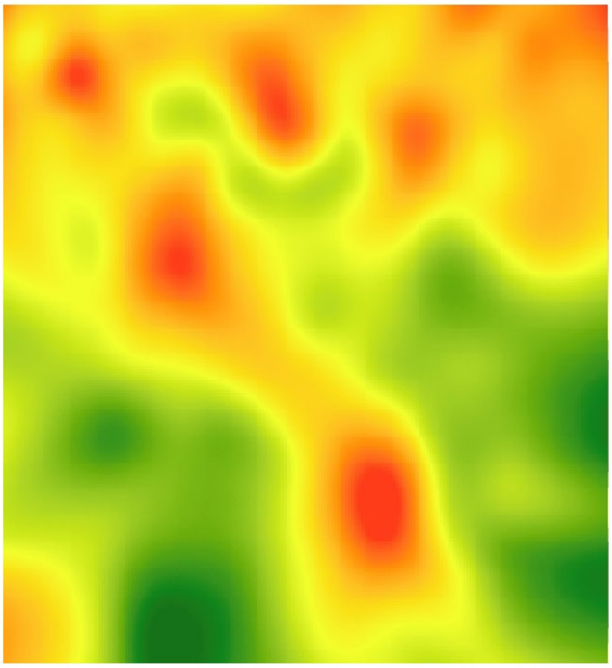


Geometric

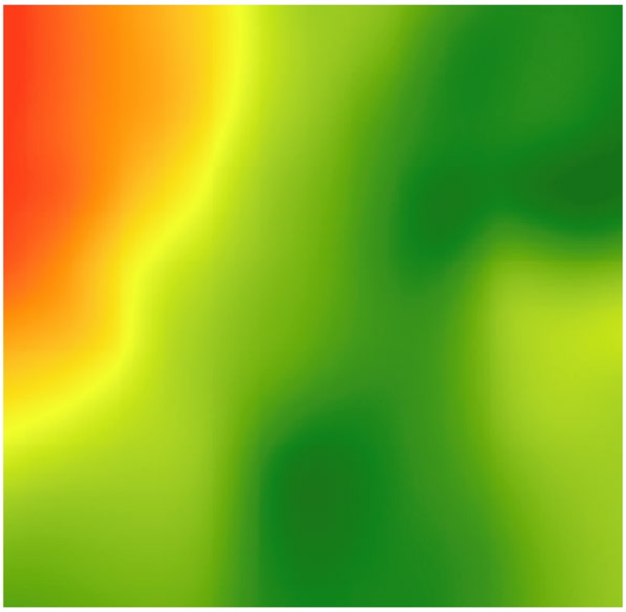
3.3. Merging Random Raster with DEM Surface

After generating the random raster with four distribution methods, it can be merged with a DEM surface using 'Raster Calculator' tool.

The combination of DEM surface and random raster distributed by 'uniform' method:



Sample 1



Sample 2

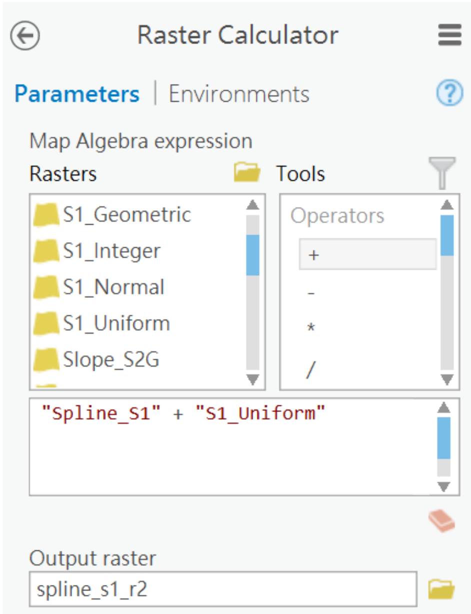
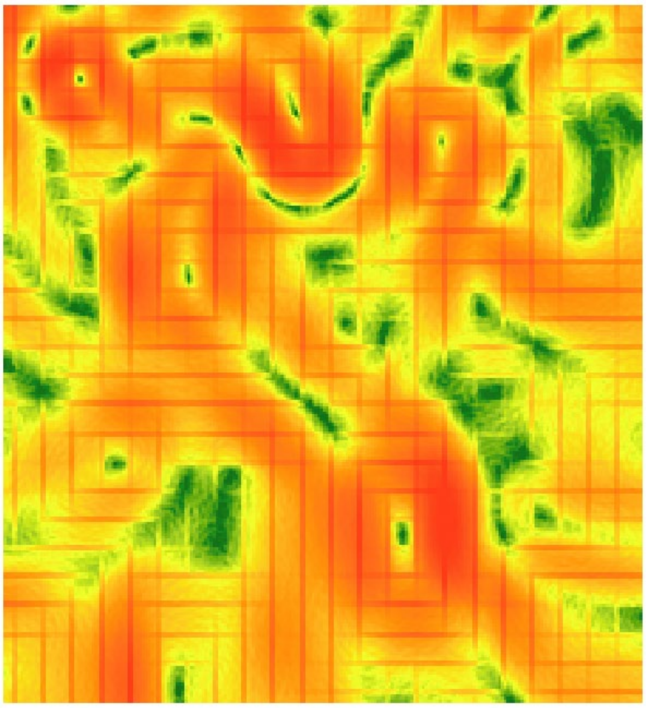


Figure 13. Add a DEM surface and a random raster in the algorithm to merge them together.

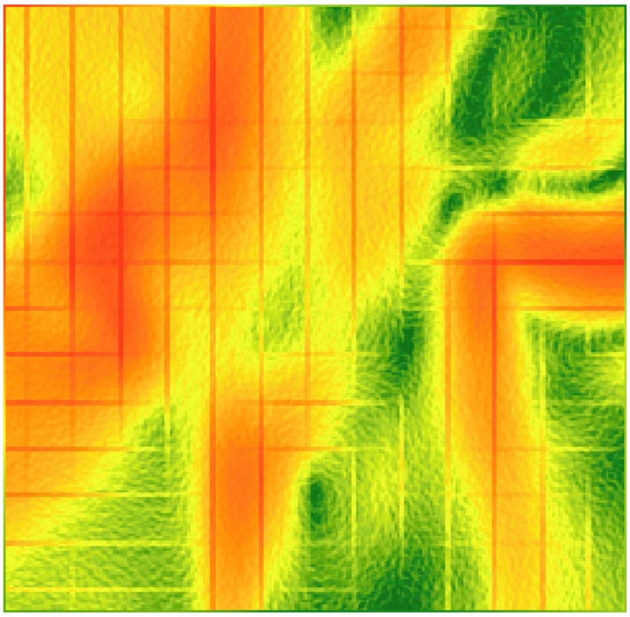
3.4. Generation of Slope Map with Noise

The DEMs are now combined with random noise of different distribution methods. Then, slope raster can again be generated by the 'slope' tool.

The resulted maps of slope with errors:



Sample 1



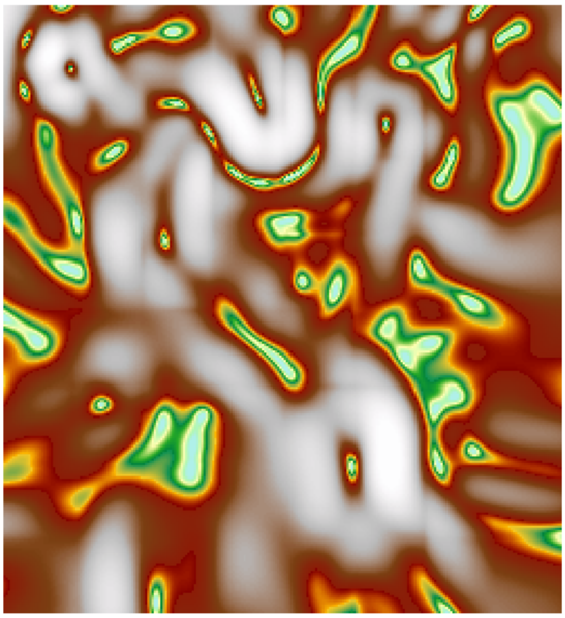
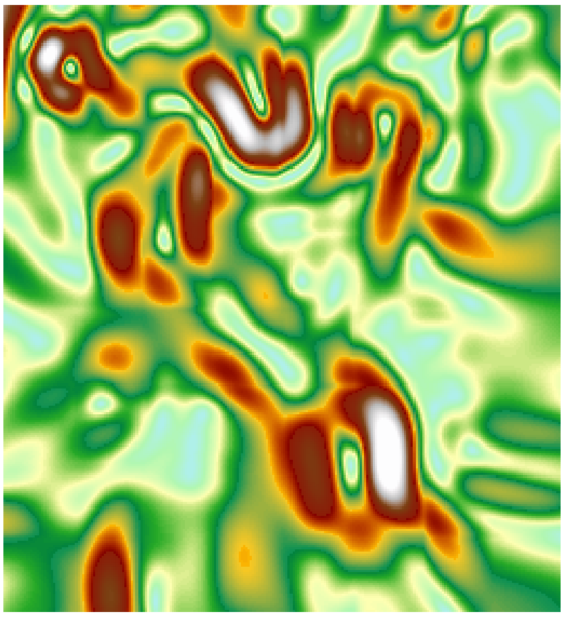
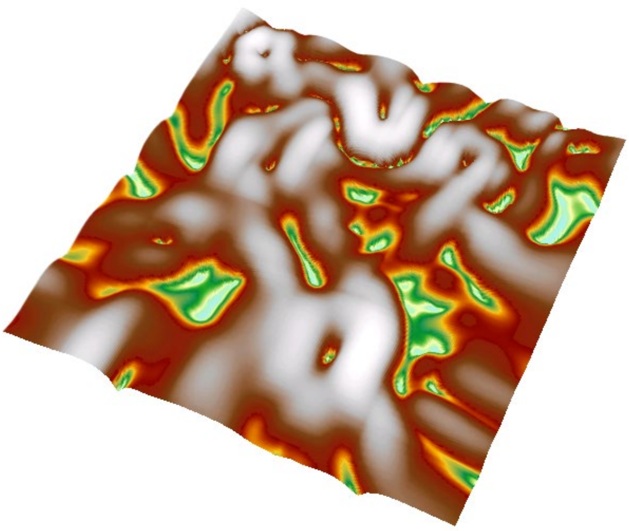
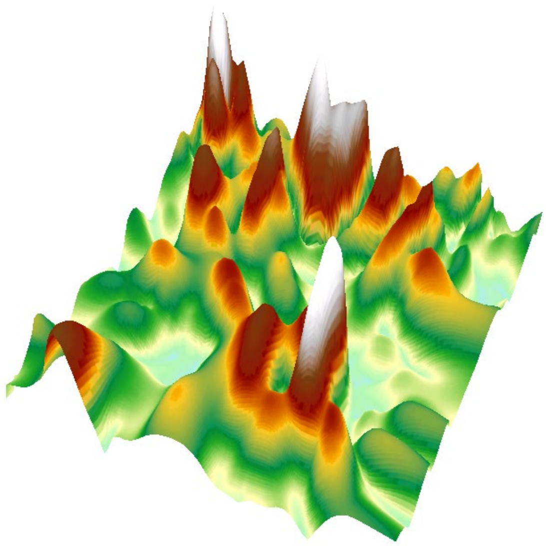
Sample 2

4. Result Analysis

4.1. Comparison of Units and Methods of Slope Calculation

4.1.1. Comparison on Calculation Unit

Slope map of Sample 1 is used for comparing units of slope calculation.

2D View	
 Degree	 Percentage
3D View	
 Degree	 Percentage

Legend	
Value 81.9802 0.59717 Degree	Value 709.755 1.0423 Percentage

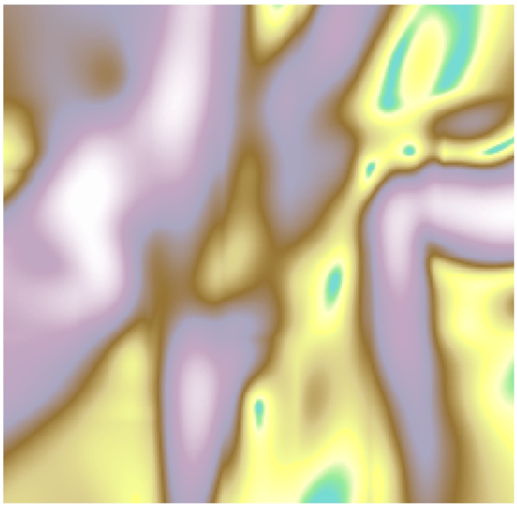
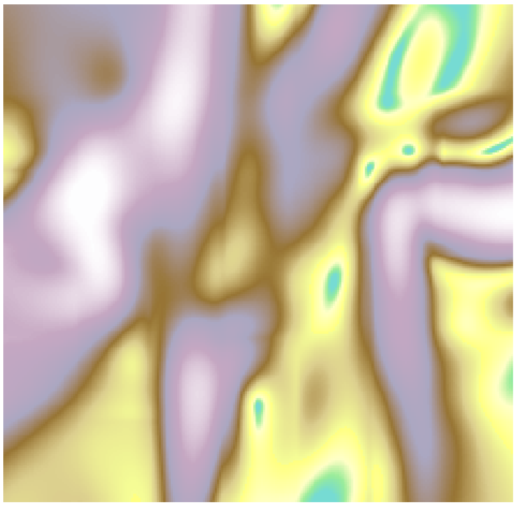
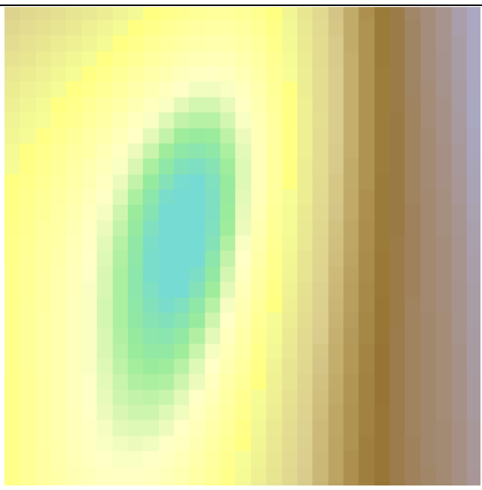
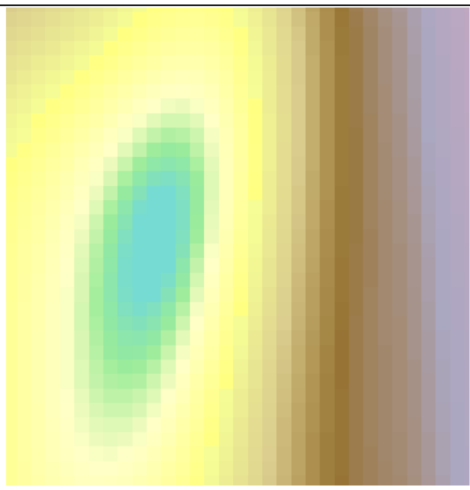
Slope maps calculated in degree and percentage look different. The reason behind is because degree calculates purely the inclination angle of the slope, and percentage calculates the ratio between the rise and run of the slope.

In 2D view, the difference between two calculation unit is illustrated by colors. In the slope with degree as unit, slopes with greater angle are represented by red and those with smaller angle are shown by green. Many steep slope as more red regions can be observed on the map. In the slope map calculated by percentage, gentler slope is displayed in green while steeper slope is displayed in red. The red pixels in the map reveals that the slopes are extremely steep as they have percentage of around 700.

In 3D view, the difference is obvious. In the map of slope calculated in percentage, steep slopes are exaggerated because the percentage values are much higher than the degree values. It can be observed that the elevation of regions in red is extremely steep.

4.1.2. Comparison on Calculation Method

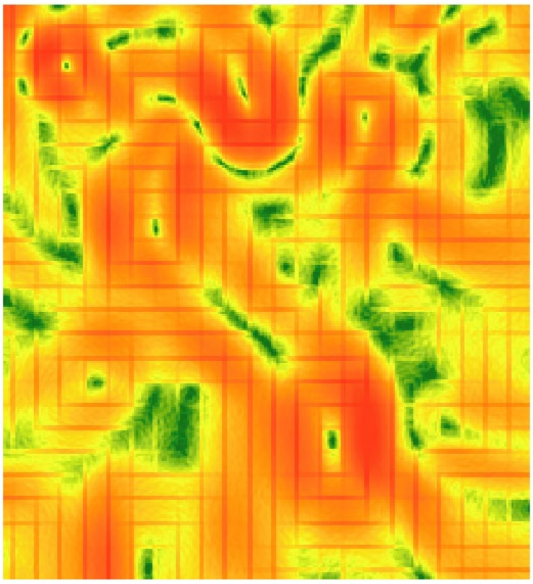
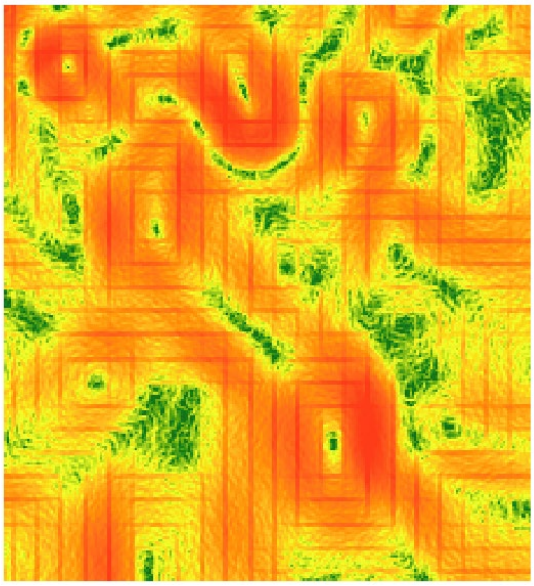
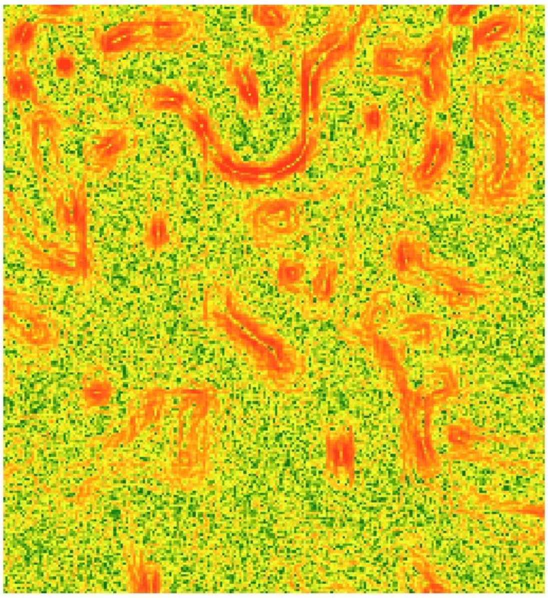
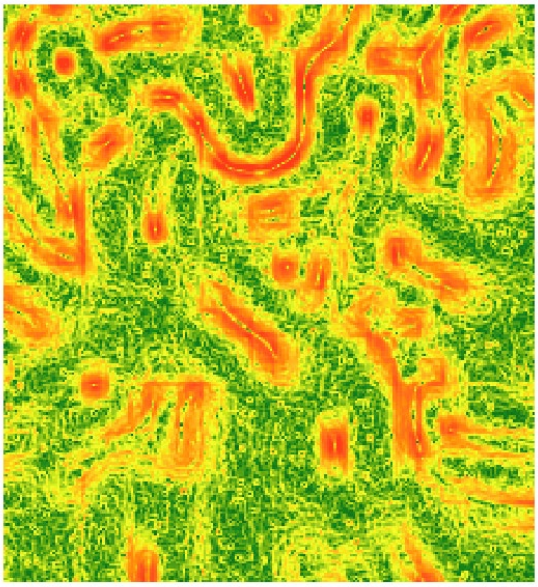
Slope map of Sample 2 is used for comparing methods of slope calculation.

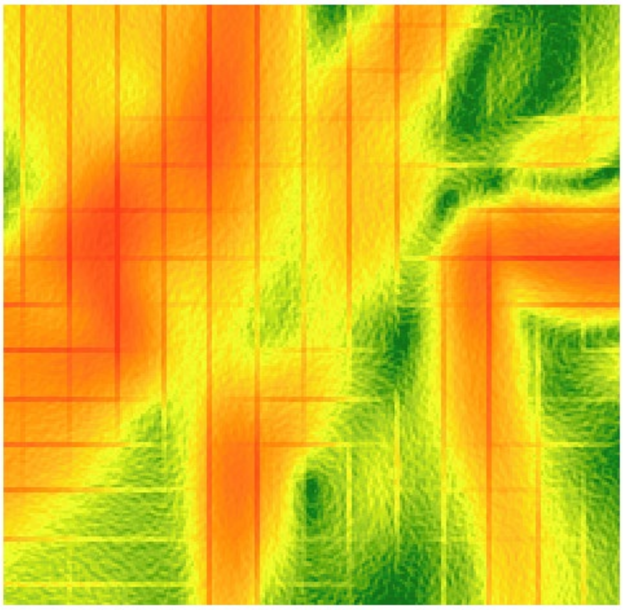
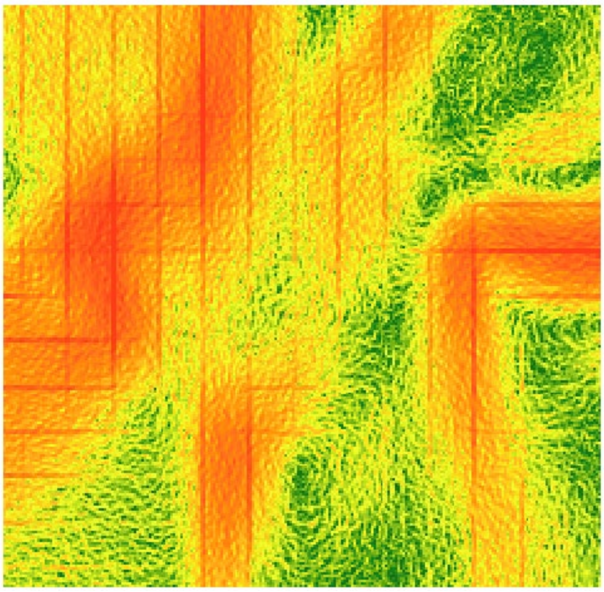
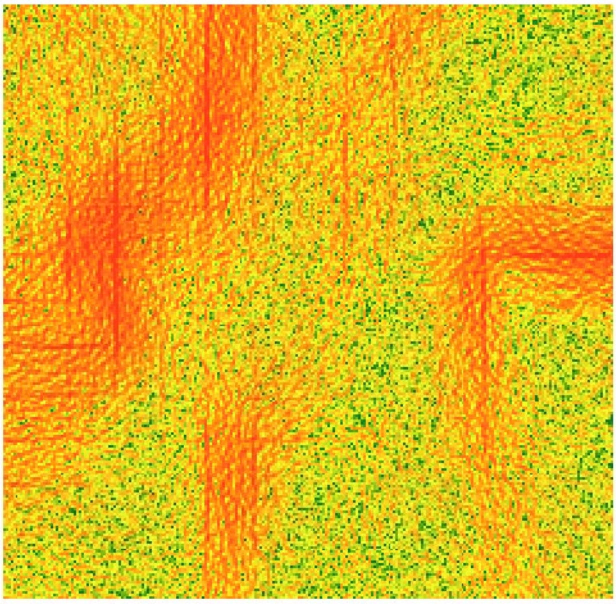
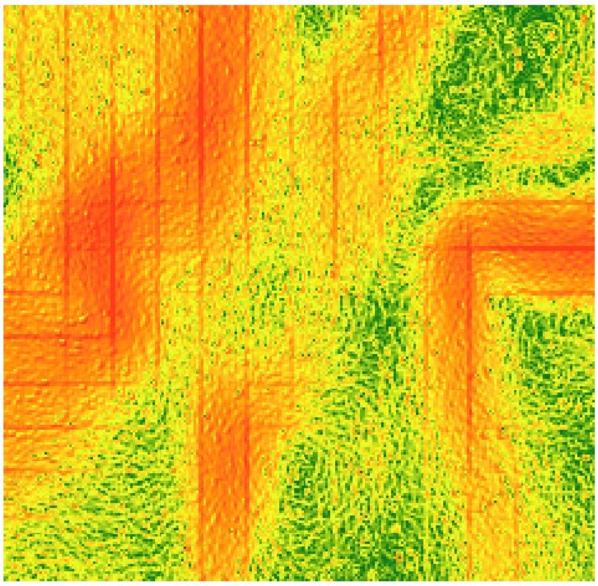
 Planar	 Geodesic
 Planar	 Geodesic

Visually, slope maps calculated by planar and geodesic methods are identical. No graphical difference can be observed even when zooming in the two slope maps. The only difference is the calculation method – planar uses a projected flat plane for calculation, while geodesic uses an ellipsoidal curve for calculation. Theoretically, calculation using geodesic mode should consider the curvature of ellipsoid. However, as the subject area is small, the effect of curvature is minimized in this case.

4.2. Effect of Errors of DEM on calculating Slope

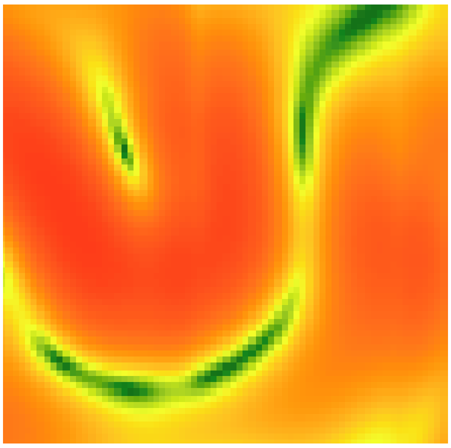
Referred to Section 3.2, random raster in 'uniform', 'normal', 'integer' and 'geometric' distribution were generated. They were then merged into the DEM surface for slope calculation. Below are the results of slope map created with random noise.

Sample 1	
 Uniform	 Normal
 Integer	 Geometric

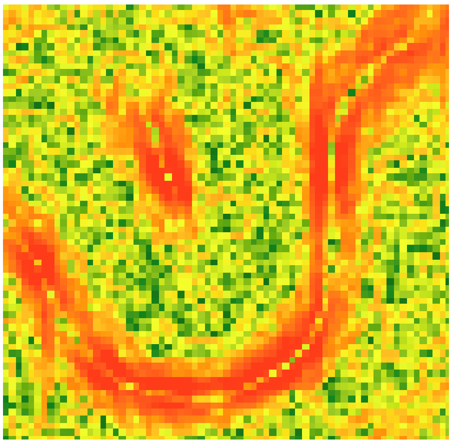
Sample 2	
 Uniform	 Normal
 Integer	 Geometric

In general speaking, the random raster has great impact on slope calculation as it is obvious to observe that the slope maps do not look similar to the original ones after noise was added on them. More variations of the slope were resulted as noise disturbs the calculation of elevation and makes the calculation inaccurate.

Take slope map of Sample 1 as example. Below shows the difference between the original slope map and the one with random raster in 'integer' distribution. Under the color representation of slope value (red for larger value, green for smaller value), value for steep slope cannot be calculated in the right-hand side map because 'integer' distribution noise interrupts the calculation of slope value.



Original map



Map in 'integer' distribution

For slope maps generated with 'uniform' and 'normal' distributed random noise, slope calculation is relatively less affected by the noise. This maybe because 'uniform' and 'normal' are continuous type of distribution that outcome of one value is related to the last value. The slope maps can generally resemble the original ones, though small differences caused by the noise can also be observed.

For slope maps generated with 'integer' and 'geometric' distributed random noise, the disturbance on slope calculation is much severe. This is because the values of random raster created by these two methods is discretely distributed. They do not follow any continuous sequence or pattern that values of the neighbouring cells from one cell can be any value. This greatly interrupts the calculation of slope and results in wrong slope values.

4.3. Effects of Colour Schemes on Indicating Slope Gradient

4.3.1. Deriving Slope Value from Slope Map

One of the greatest advantages of digital terrain modeling is that users can always get information of a terrain surface just by a few clicks. In this practice, slope value can also be derived from the 'attribute' of the grid. In the former parts, slope is generated in pixel format. Then, slope value can be found by clicking in a certain cell.

When zooming in the slope map, users can see detailed colour change of the cells as the slope gradient differs. The below shows the extraction of slope value from the slope map.

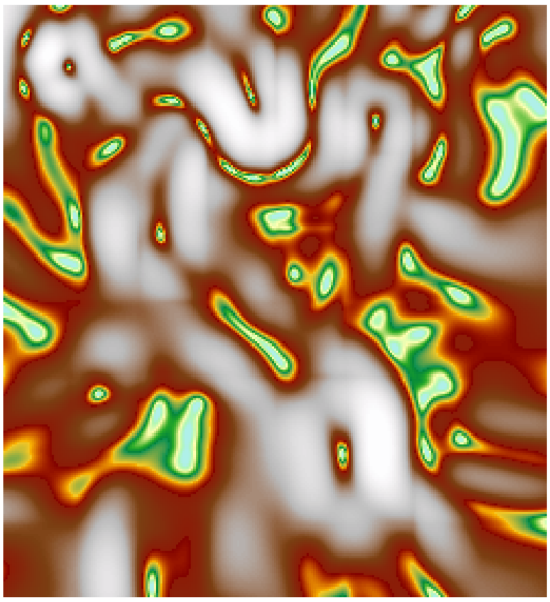
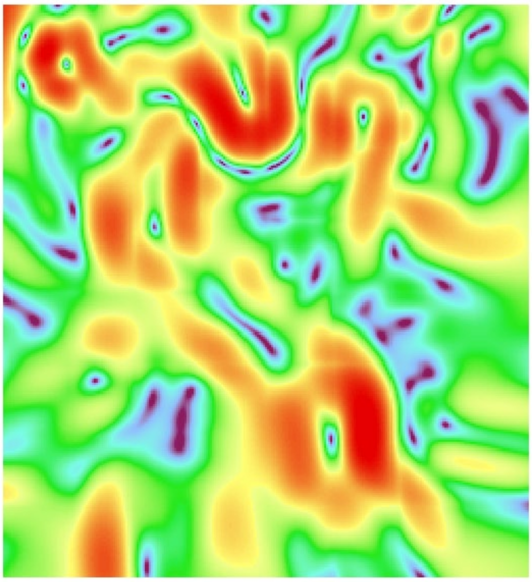
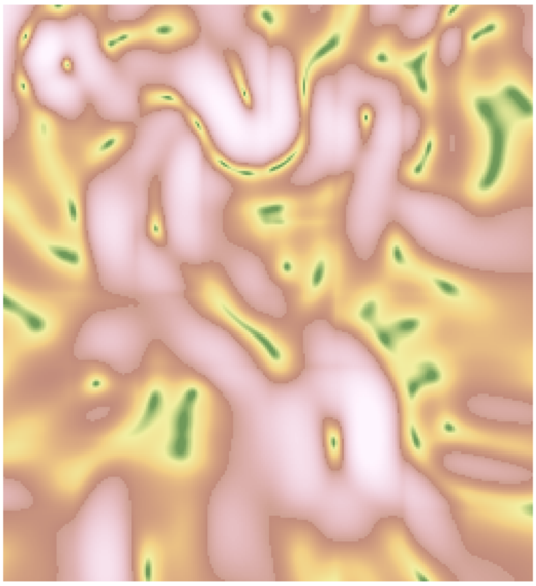
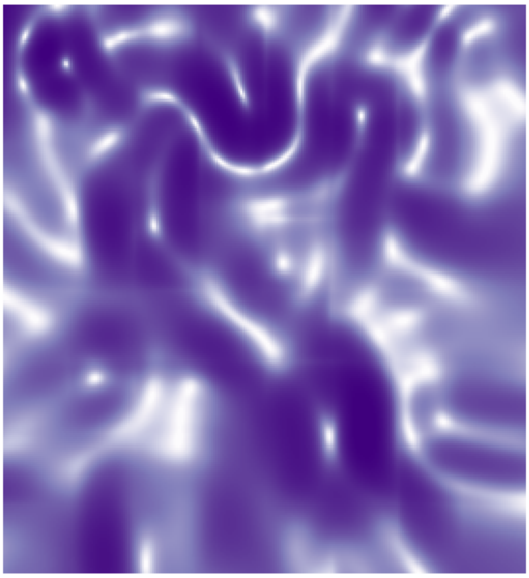
Stretch.Pixel Value	28.254494
Stretch.Stretched value	90

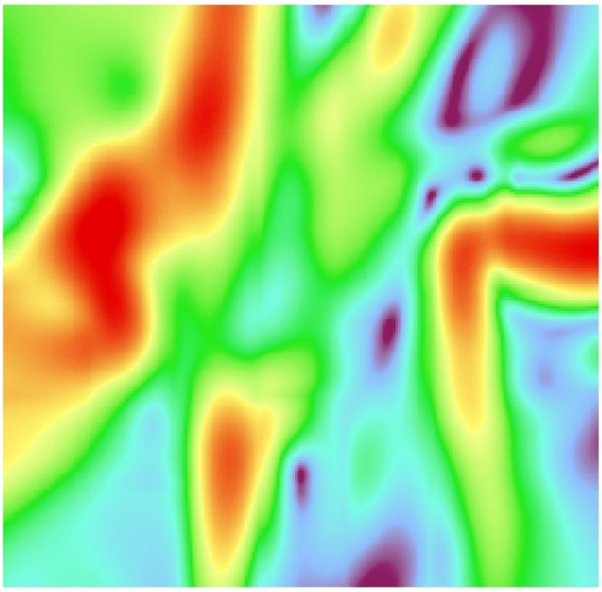
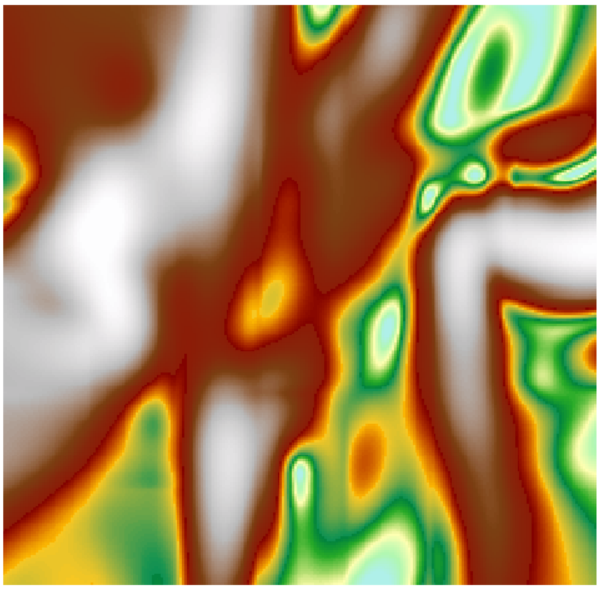
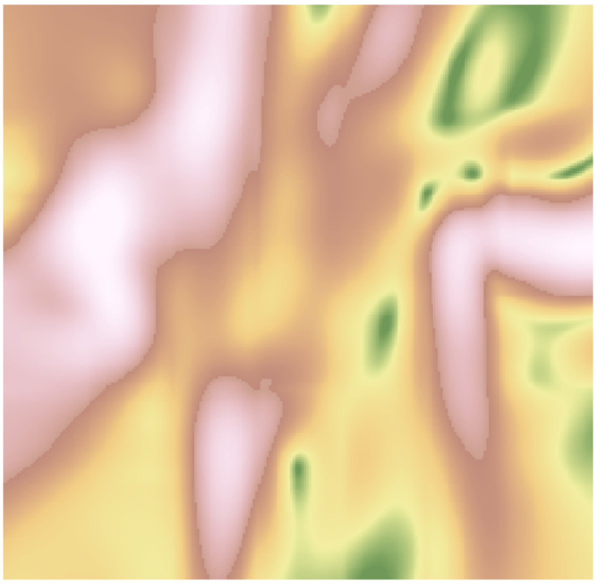
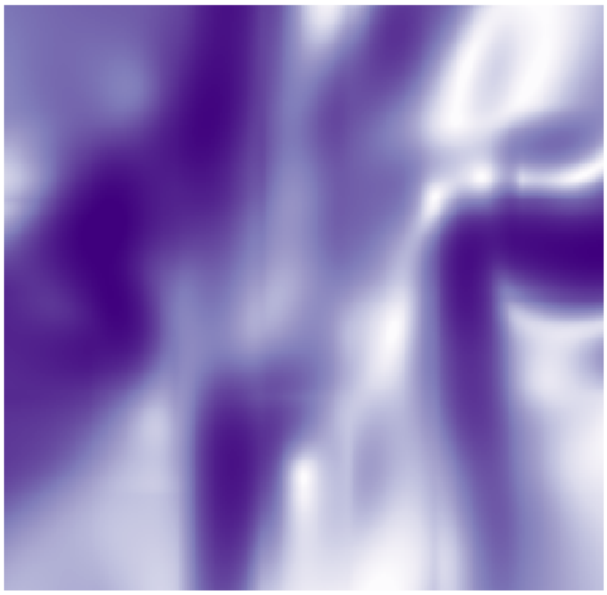
Stretch.Pixel Value	34.124035
Stretch.Stretched value	113

It illustrates that users can interpret change in slope not only by change in cell colour, but also by difference in cell value.

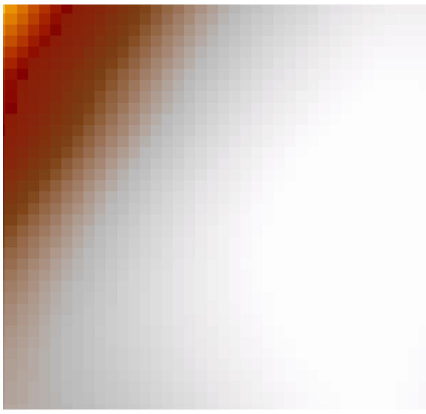
4.3.2. Comparison of Colour Schemes

A 'good' colour scheme should be attractive to audience and it must also support the message of the map to match the nature of the data (Harrower and Brewer, 2003). In this practice, 'Elevation #1', 'Bathymetric', 'Surface' and 'Purples (Continuous)' were used to demonstrate how colour scheme affects the interpretation of slope value on a map.

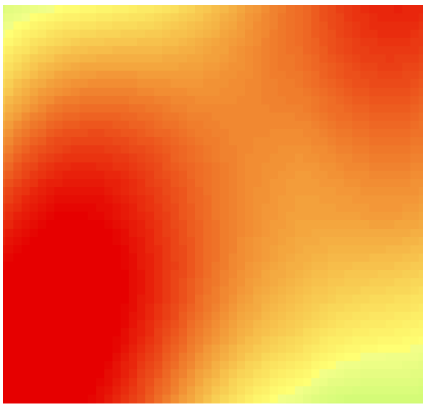
Sample 1	
 Elevation #1	 Bathymetric
 Surface	 Purples (Continuous)

Sample 2	
 Bathymetric	 Elevation #1
 Surface	 Purples (Continuous)

As mentioned in Lab 7, colour scheme is divided into three categories – sequential and diverging for showing continuous data, and qualitative for showing discrete data. In this case, value in a slope map is a continuous data as it shows the changes in elevation of a terrain. Therefore, theoretically sequential and diverging colour schemes should be employed on slope map, From the above colour schemes, 'Purples (Continuous) is a sequential colour scheme that is supposed to fit the nature of data type to show the change in slope value of the terrain. However, the change in slope is not distinct enough as the purple colour looks similar. As the below comparison (from Sample 2) shows, 'Purples (Continuous)' may be less capable on showing slope change compared with 'Elevation #1'. However, 'Purples (Continuous)' can indeed show the gradual change in inclination though the change is not obvious.

	
Purples (Continuous)	Elevation #1

For 'Bathymetric' and 'Elevation #1', they can clearly show the change in slope as the colour contrast is distinct. Detailed fluctuations in slope change can be easily observed. As demonstrated below, small change in slope is represented in red and orange in 'Bathymetric', and in white and grey in 'Elevation #1'.

	
Bathymetric	Elevation #1

5. Discussion – Employ Slope Values to Conduct Hydrology Analysis

In rainy seasons in Hong Kong, landslide and flooding occur in some area where slope is not properly maintained. Slope surface drainages plays an important role in preventing erosion and improving slope stability (CEDD, 2013). Slope surface drainage should be properly managed to prevent such natural hazards to happen. Slope value model can assist with the assessment of slope surface runoff.

For exapmle, runoff coefficient (C) can be calculated using slope gradient value. Runoff coefficient is the ratio of surface runoff to rainfall depending on the land use and the gradient (i.e. slope value) of the ground. The coefficient includes empirical factors that cannot be determined precisely. The Drainage Service Department suggests that C for undeveloped grassland should fall from 0.05 to 0.35 (as rainfall would be infiltrated by healthy soil), for steep natural slope should fall from 0.4 to 0.9 (as steep slope would lead to greater surface runoff), and for urban area should be 1.0 (as most area are covered by man-made structure like concrete).

Then, time of concentration (tc) can be derived as the time needed for water to flow overland from the most remote point in a catchment area to its outlet. Peak surface runoff occurs when the duration of the design rainfall with a constant intensity is equal to the concentration time of the catchment.

$$t_c = \frac{0.14465L}{H^{0.2}A^{0.1}} \quad \text{.......} \quad (2.2)$$

where
t_c = time of concentration (in min)
A = catchment area (in m^2)
H = average slope (in m per 100 m), measured along the line of natural flow, from the summit of the catchment to the point under consideration
L = distance (on plan) measured along the line of natural flow between the summit and the point under consideration (in m)

Figure 14. Equation to calculate time of concentration (CEDD).

With the time of concentration computed by slope gradient value, we can determine the time and volume of immediate discharge in the catchment in response to a rainfall event. It allows related authorities to prepare better for the coming of rainy season and design a more effective drainage system in the catchment areas.

6. Conclusion

The generation of DEM can be used to further investigate slope value by creating a slope map for the terrain. In this practice, slope maps were generated under normal circumstance and with random noise. It is found that random noise places great disturbance on slope calculation as the resulted slope map can be completely different from the original one. Besides, colour scheme can affect one's interpretation on slope gradient. Therefore, an appropriate colour scheme should be chosen according to the nature and purpose of the slope map to facilitate users' interpretation of the map. From additional research, it is suggested that slope value can be employed to estimate surface runoff of slope for the assessment of flood. This can help related departments to implement effective policies and build structures to minimize the impact brought by natural hazards.

Reference

ArcGIS Pro Help. Esri. Distributions for assigning random values.

https://pro.arcgis.com/en/pro-app/tool-reference/data-management/distributions-for-assigning-random-values.htm

ArcGIS Pro Help. Esri. Create Random Raster (Data Management).

https://pro.arcgis.com/en/pro-app/tool-reference/data-management/create-random-raster.htm

M., Harrower. & C.A., Brewer. (2003). Tool for Selecting Colour Schemes for Maps. Retrieved from: https://www.semanticscholar.org/paper/Tool-for-Selecting-Colour-Schemes-for-Maps-Harrower-Brewer/8c9af5bd12b36e450ba564f644008b871dba5cdf

Civil Engineering and Development Department, HKSAR. (2013). Review of Methods in Estimating Surface Runoff from Natural Terrain. Retrieved from: https://www.cedd.gov.hk/filemanager/eng/content_514/er292links.pdf