LSGI3242A DIGITAL TERRAIN MODELLING

LAB 8

EFFECT OF DEM ACCURACY ON SLOPE VALUES

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CHAPTER 1
INTRODUCTION

1.1 Background

Morphometric terrain parameters are those that can be derived directly from the DTM using some local operations, such as slope and aspect, complexity index, and so on. Slope is the fundamental attribute of a terrain point. In this context, slope information can probably serve as the basis for the various application, e.g. error detection. Often the presence of errors will distort the image, i.e. the appearance, of the spatial variation present in DTM data sets much more seriously than that resulting from random noise. In some cases, totally undesirable results may be produced in the DTM and in the products derived from it, due to the existence of such errors. Therefore, methods are needed to detect this type of errors in DTM data set and to ensure their removal from the data set by the use of slope information.

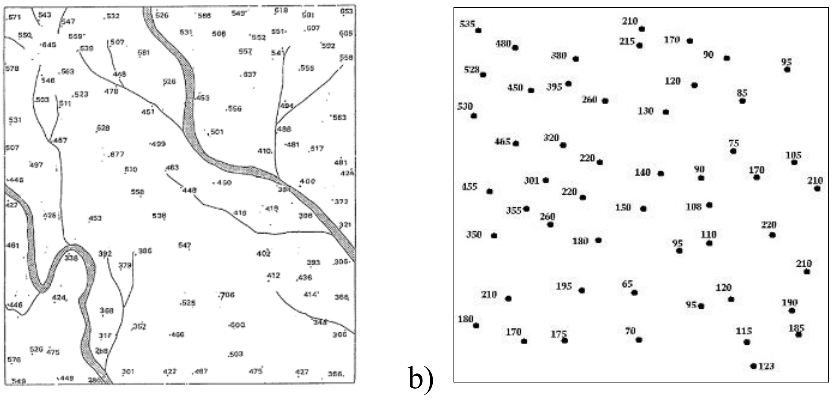
1.2 Objective of the report

This report is to explore the effect of random errors on slope values by performing slope calculation to generate slope map based on DEM and adding random noise manually. With the resulting maps, the comparison between the methods of calculating slope, the effect of errors of DEM on calculating slope by creating a raster dataset of random values with different distribution, and the effect of colour schemes on indicating the gradient of extracted slope values are explored. In order to extend the discussion, the detection methods of gross error in raster-based DEM using slope information are also be discussed.

CHAPTER 2
METHODS AND MATERIALS

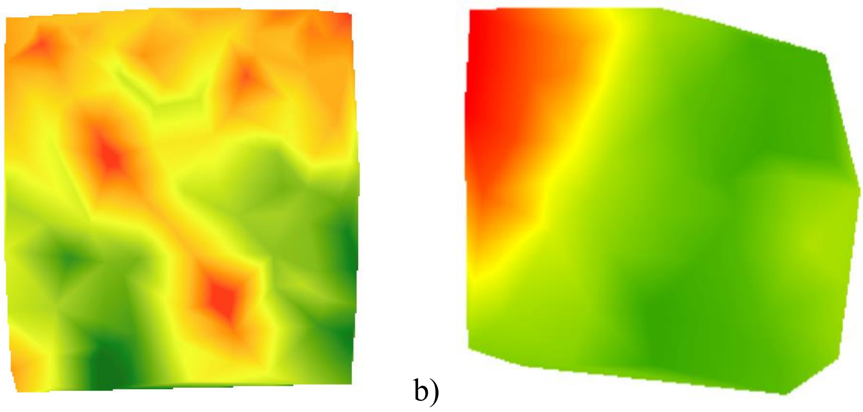
2.1 Introduction

In this lab, two spot height maps (sample 1 and sample 2 shown as figure 2.1) were used to perform slope calculation. As mentioned in lab 4, a triangulated irregular network (TIN) can serve as a basis for precise calculations since it is a vector-based representation of a continuous surface consisting entirely of triangular facets. Compared to the raster-based DEM generated by the interpolation method, TIN preserve abrupt linear features and are excellent for calculations of slope, aspect, and surface area. Hence, the raster DEMs derived from TIN (shown as figure 2.2) were used to perform slope calculation in the experiment in order to demonstrate the effect of error obviously.



a) b)

Figure 2.1 Digital spot height maps, a) sample 1, b) sample 2



a) b)

Figure 2.2 The raster DEMs derived from TIN, a) sample 1, b) sample 2

In order to investigate the influence of errors on slope calculation, a random distributed raster dataset is generated as errors. Then the random grid is merged with the sample terrain model to output the DEM with errors. The slope maps with errors can then be generated. To demonstrate the effect of errors in different distribution,

1) Uniform (Maximum: 1.0, Minimum: 0.0),
2) Normal (Mean: 0.0, Standard Deviation: 1.0),
3) Exponential (Mean: 1.0),
4) Possion (Mean: 1.0),
   and
5) Gamma (Alpha: 1.0, Beta: 1.0),

distributions were employed to create an error raster of random values.

In next part, the principles of slope calculation are discussed in detail.

2.2 Overview of the principle of slope calculation

Slope is the first derivative of a surface and has both magnitude and direction. That is, slope is a vector consisting of gradient and aspect. The Slope tool provided in ArcGIS Pro can identifies the steepness at each cell of a raster surface. The lower the slope value, the flatter the terrain; the higher the slope value, the steeper the terrain. The defination of slope are as follows:

The surface function is:

$$z = f(x, y)$$

Then, the slope is defined as:

$$Slope_x = \frac{df}{dx} = f_x$$

$$Slope_y = \frac{df}{dy} = f_y$$

Slope can be derived from the raster-based DEM using simple local operations. Compared to TIN, many approaches are available to compute slope and aspect from a raster-based DEM. In this report, only the method implemented in ArcGIS Pro are presented. Figure 2.3 is a window with nine cells from a grid DEM. From this window, the slope values of the central cell, that is, with height z0, can be estimated as follows:

$$Slope = \tan\alpha = \sqrt{Slope_{Row}^2 + Slope_{col}^2}$$

In these formulae, $Slope_{Row}$ and $Slope_{col}$ are the slopes in the row and column directions, respectively.

z_5	z_2	z_6
z_1	z_0	z_3
z_8	z_4	z_7

Figure 2.3 A moving window for the computation of slope and aspect value (Li, et al., 2005)

In ArcGIS Pro, there is two methods to compute slope value of each cell. Both of them are performed under a frame of using a 3 by 3 cell neighborhood (moving window) as mentioned above. The detailed algorithm for computing slope value will be discussed in chapter 5.1. However, it should be noted that, for each neighborhood, if the central cell is NoData, the output is NoData. The computation also requires at least seven cells neighboring the processing cell have valid values. If there are fewer than seven valid cells, the calculation will not be performed, and the output at that processing cell will be NoData. The cells in the outermost rows and columns of the output raster will be NoData. This is because along the boundary of the input dataset, those cells do not have enough valid neighbors.

2.3 Materials

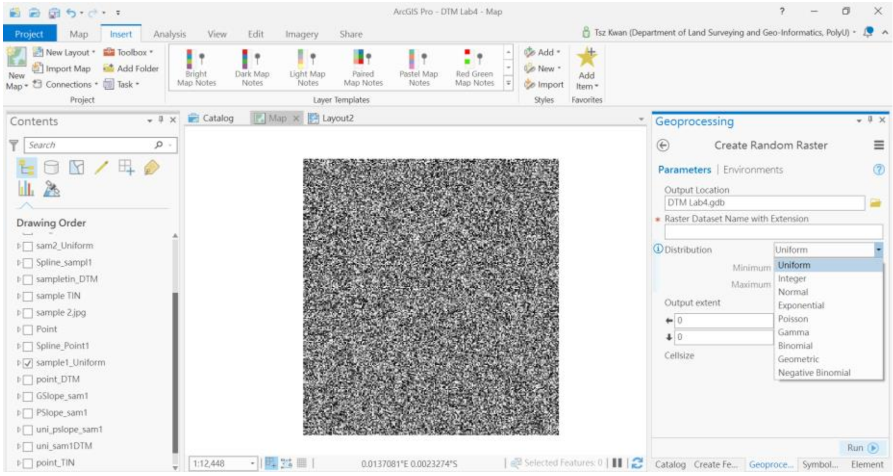
The objectives pursued was to investigate the effect of random errors on slope values. Therefore, the materials required will be as follows:

- ArcGIS Pro
- Raster-based DEM of sample 1 and sample 2 derived from TIN

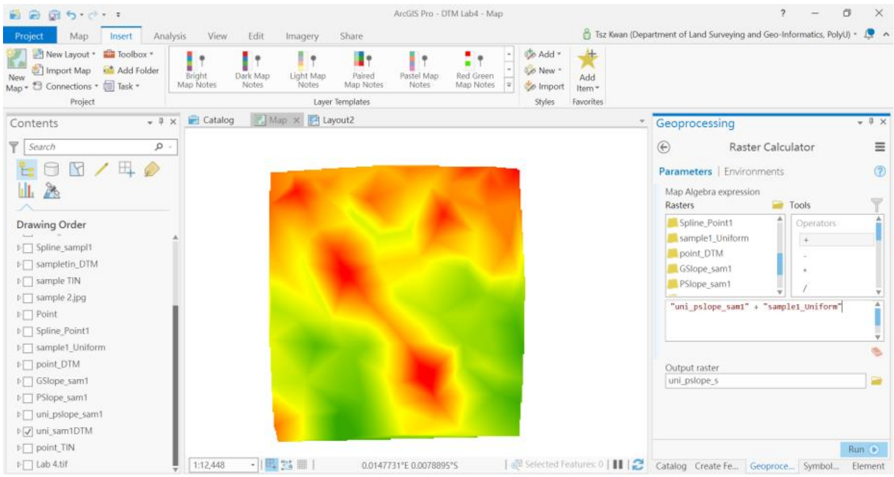
CHAPTER 3
EXPERIMATAL PROCEDURE

3.1 Generate random rasters and merging multiple rasters

To create a raster dataset with a random value as an error, in ArcGIS Pro, "Create Random Raster" function is employed. In this experiment, all the available distributions were used to create random values to investigate the effect of error. In "Output extent", select the appropriate layer to create a random raster of the same size as the DEM in sample 1 and 2.



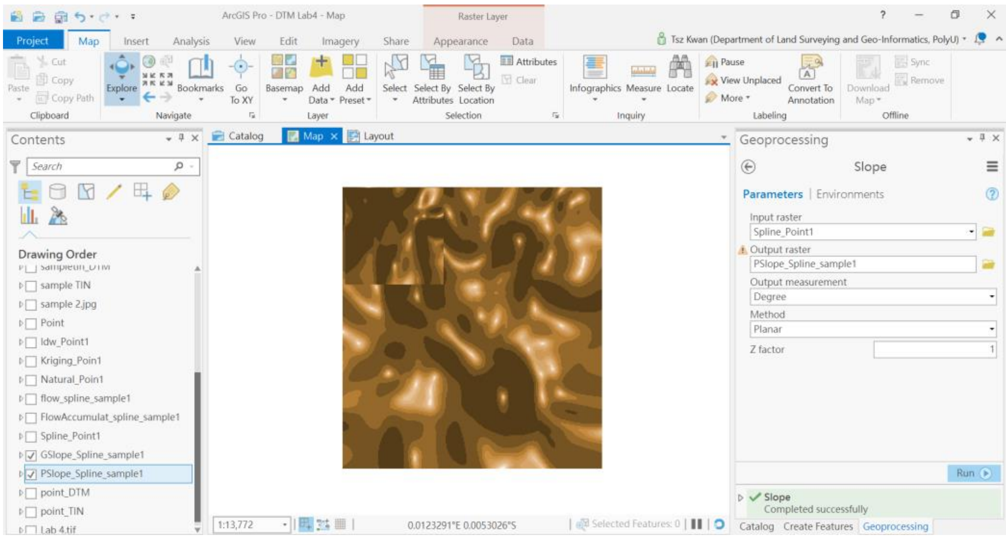
To merge the raster-based DEM with random noise, "Raster Calculator" function is employed. The Raster Calculator tool can be used to create and execute a Map Algebra expression that will output a DEM with random noise that create in last step.



Repeat the whole process for generating the raster-based DEMs with random noise in different distribution.

3.2 Perform slope calculation based on DEM

To identifies the steepness at each cell of a raster surface, “Slope” function is employed to generate slope map. The output slope raster can be calculated in two types of units, degrees or percent (percent rise). In this experiment, set “Degree” as the output measurement for better understanding.



Repeat the whole process again for calculating the slope in planar and geodesic methods from sample 1’s and sample 2’s DEM with random noise.

CHAPTER 4
RESULTS

In this session, the DEM and the slope map with gamma error are shown as follows. The other resulting map of demonstrating different methods, i.e. distribution and projection methods, will be shown and discussed in detailed in the next session.

4.1 Sample 1

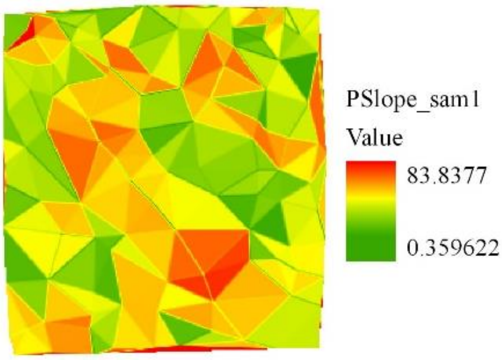


Figure 4.3 The slope map in planer method without error (slope colour scheme)

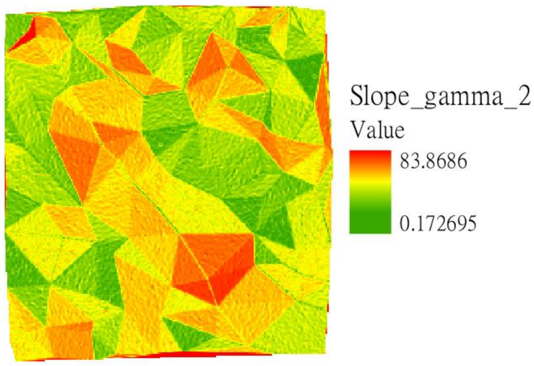


Figure 4.1 The slope map in planer method with gamma error (slope colour scheme)

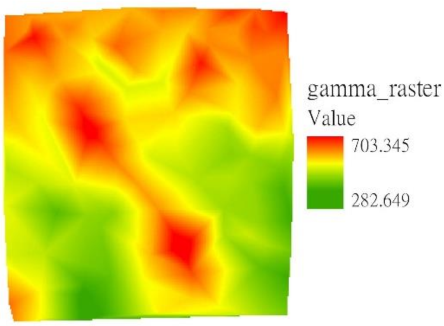


Figure 4.2 The DEM with gamma error
(slope colour scheme)

4.2 Sample 2

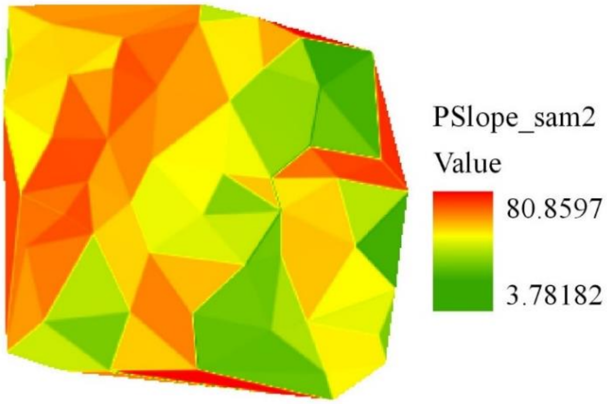


Figure 4.3 The slope map in planer method without error (slope colour scheme)

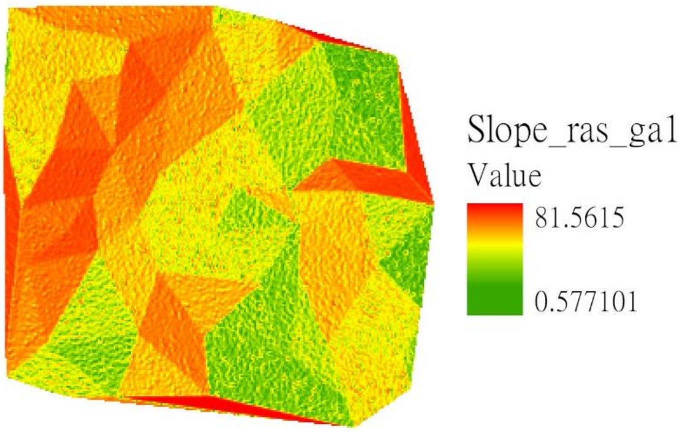


Figure 4.3 The slope map in planer method with gamma error (slope colour scheme)

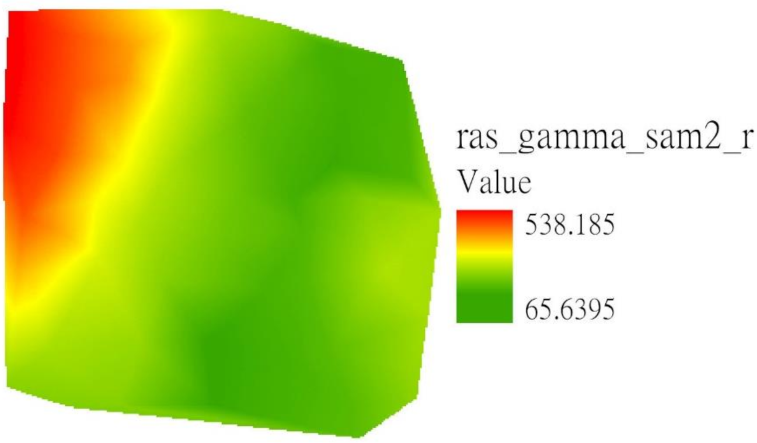


Figure 4.4 The DEM with gamma error
(slope colour scheme)

CHAPTER 5
ANALYSIS

5.1 The methods of calculating slope

The Slope tool identifies the steepness at each cell of a raster surface. The lower the slope value, the flatter the terrain; the higher the slope value, the steeper the terrain. In addition, ArcGIS Pro provides two methods, planer and geodesic, for slope computation. Both planar and geodesic computations are performed using a 3 by 3 cell neighbourhood (moving window). However, the two methods use different dimensions in the coordinate system to calculate slope. In this session, the principle and algorithm of two methods are introduced and are evaluated.

5.1.1 Planar method

For the planar method, the slope is measured as the maximum rate of change in value from a cell to its immediate neighbours. Basically, the maximum change in elevation over the distance between the cell and its eight neighbours identifies the steepest downhill descent from the cell. The calculation is performed on a projected flat plane using a 2D Cartesian coordinate system. The slope value is calculated using the average maximum technique.

Planar slope algorithm

The values of the centre cell and its eight neighbours determine the horizontal and vertical deltas. The neighbours are identified as letters from a to i, with e representing the cell for which the aspect is being calculated.

a	b	c
d	e	f
g	h	i

Figure 5.1 Surface window (esri, n.d.)

The rate of change in the x direction for cell e is calculated with the following algorithm:

$$\left[\frac{dz}{dx}\right] = \frac{\left((c + 2f + i) \times \frac{4}{wght1} - (a + 2d + g) \times \frac{4}{wght2}\right)}{(8 \times x_{cellsize})}$$

The rate of change in the y direction for cell e is calculated with the following algorithm:

$$\left[\frac{dz}{dy}\right] = \frac{\left((g\ +\ 2h\ +\ i) \times \frac{4}{wght3} -\ (a\ +\ 2b\ +\ c) \times \frac{4}{wght4}\right)}{(8\ \times y_{cellsize})}$$

where wght1 and wght2 are the horizontal weighted counts and wght3 and wght4 are the vertical weighted counts of valid cells.

Taking the rate of change in the x and y direction, the slope for the centre cell e is calculated using the following:

$$slope\_degrees\ =\ arctan(\sqrt{\left(\left[\frac{dz}{dx}\right]^2\ +\ \left[\frac{dz}{dy}\right]^2\right)}) \times \frac{180}{\pi}$$

5.1.2 Geodesic method

With the geodesic method, the calculation will be performed in a geocentric 3D Cartesian coordinate system by considering the shape of earth as an ellipsoid. The slope value is calculated by measuring the angle between topographic surface and the referenced datum.

Geodesic slope algorithm

To begin with, the surface raster has to transformed from the input coordinate system into a 3D geocentric coordinate system. The geodesic computation uses an X, Y, Z coordinate that is calculated based on its geodetic coordinates (latitude φ, longitude λ, height h). If the coordinate system of the input surface raster is a projected coordinate system (PCS), the raster is first re-projected to a geographical coordinate system (GCS) where each location has a geodetic coordinate, and then transformed to the Earth Centred, Earth Fixed (ECEF) coordinate system. The height h (z-value) is the ellipsoid height referenced to the ellipsoid surface. The transformation of two coordinate system is calculated with the following algorithm:

$$X\ =\ (N(\varphi) + h)cos\varphi cos\lambda$$

$$Y\ =\ (N(\varphi) + h)cos\varphi sin\lambda$$

$$Z\ =\ \left(\frac{b^2}{a^2} \times N(\varphi) + h\right)sin\varphi$$

where N is the radius of prime vertical

The geodesic slope is the angle formed between the topographic surface and the ellipsoid surface. Any surface parallel to the ellipsoid surface has a slope of 0. To calculate the slope at each location, a 3 by 3 cell neighbourhood plane is fitted around each processing cell using the Least Squares Method (LSM). The best fit in the LSM minimizes the sum of squared difference (dz_i) between the actual z-value and the fitted z-value. See the illustration below for an example.

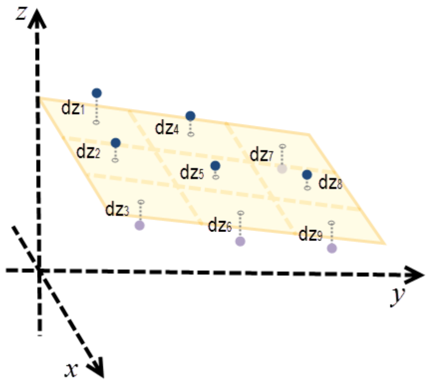


Figure 5.2 Least Squares Fitting example (esri, n.d.)

Here, the plane is represented as $z = Ax + By + C$. For each cell centre, dz_i is the difference between the actual z-value and the fitted z-value.

The plane is best fitted when $\sum_{i=1}^{9} dz_i^2$ is minimized. After the plane is fitted, a surface normal is calculated at the cell location. At the same location, an ellipsoid normal perpendicular to the tangent plane of the ellipsoid surface is also calculated.

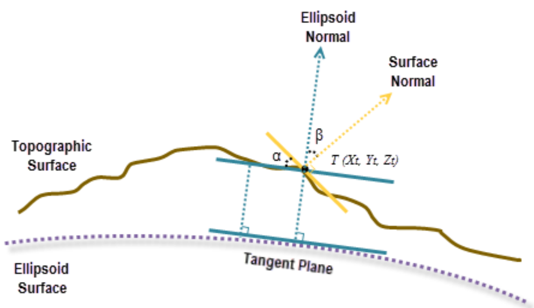


Figure 5.3 Geodesic slope computation (esri, n.d.)

The slope, in degrees, is calculated from the angle between the ellipsoid normal and the topographic surface normal, represented as β here. From the illustration above, the angle α is the geodesic slope, which is the same as angle β, pursuant to the law of congruent geometry.

5.1.3 Comparison between methods of calculating slope

With respect to the above algorithm, it is clear that the geodesic method could produce a more accurate slope than the planar method since the slope calculation perform by considering the shape of earth as an ellipsoid rather than the flat plane. However, the selection of method depends on the size of the study area. In this lab, the terrain model adopted in the experiment is large scale DEM. As can been seem from figure 5.4, whether geodesic method or plane method is adopted in the study area, the range of calculated slope of both methods only have little difference since the shape of earth in this case can be consider as the flat plane. On the other hand, figure 5.5 demonstrates the slope maps of the world's land and ocean floor derived from a small scale world DEM. It is obvious that the range of calculated slope of both methods have huge difference. In this case, the geodesic method would be appropriate because no planar coordinate system is suitable for global collections of features. Therefore, the geodesic method is an ideal method for small scale DEM requiring consideration of the curvature of the Earth. For large scale DEM, both geodesic and planner method can also be adopted.

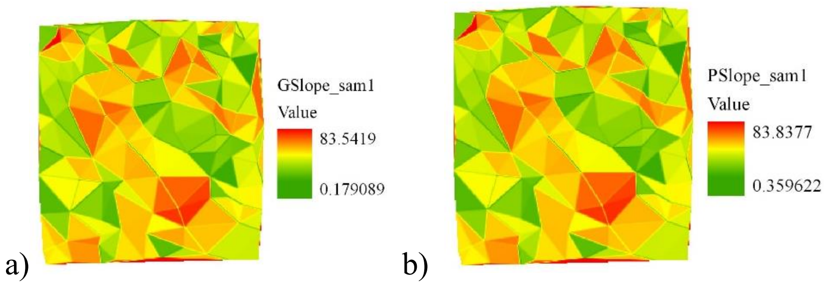


Figure 5.4 The slope maps of sample 1 in, a) geodesic, b) planer method

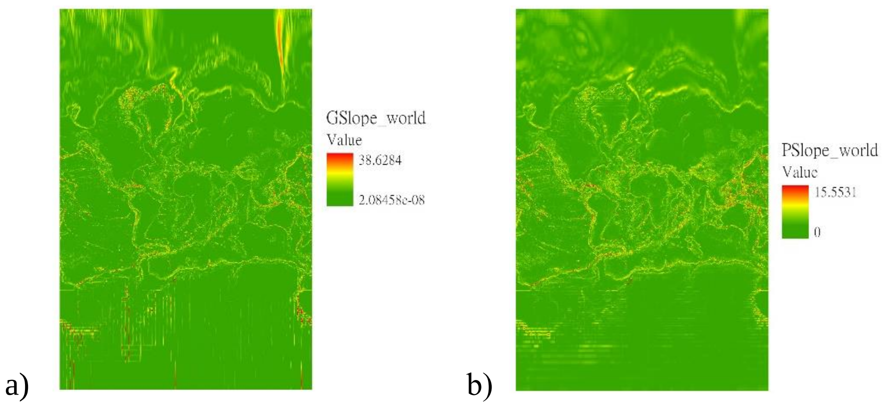


Figure 5.5 The slope maps of the world's land and ocean floor in, a) geodesic, b) planer method

5.2 The effect of errors (i.e. distribution) of DEM on calculating slope

For DEM source data acquired by photogrammetry, errors in source materials include those in aerial photographs, e.g., those caused by lens distortion and those at control points. In classic error theory, the variability of serious measurements of a single quantity is due to observational errors. Such errors do not follow any deterministic rule, thus leading to the concept of random errors. Random errors are also referred to as random noise in image processing and as white noise in statistics. Noise is very difficult to remove it from the digital images without the prior knowledge of noise model. In the experiment, the noise was added to generated elevation surfaces to simulate the occurrences of phenomena in space or time. Various types of distributions form the basis of statistical tests in which observations can be compared with observations of known distributions. In this session, the cause and the principle of noise are introduced.

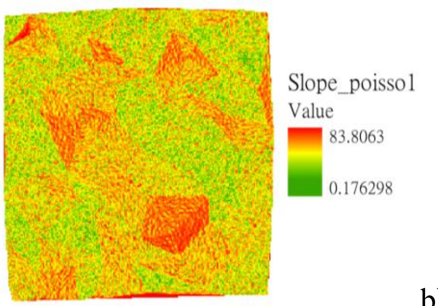
5.2.1 Gamma distribution

Gamma noise is generally seen in the laser-based images. It obeys the Gamma distribution. Gamma distribution is a continuous probability distribution. Gamma distribution models the sum of multiple independent, exponentially distributed variables. The formula for gamma distribution is as follows:

$$f(x,\alpha,\beta) = x^{\alpha-1}\frac{\beta^{\alpha}e^{-\beta x}}{\Gamma(\alpha)}$$

As can been seen in figure 5.6, with the increasing of alpha and beta, the noise of the noise map is also increased.

a)



b)

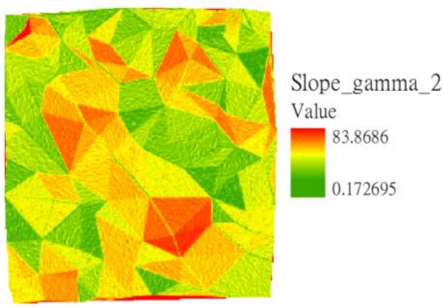


Figure 5.6 The slope map of sample 1 in Gamma distribution,
a) Alpha: 3.0, Beta: 3.0, b) Alpha: 1.0, Beta: 1.0

5.2.2 Poisson distribution

The appearance of this noise is seen due to the statistical nature of electromagnetic waves such as x-rays, visible lights and gamma rays. The x-ray and gamma ray sources emitted number of photons per unit time. These rays are injected in patient's body from its source, in medical x rays and gamma rays imaging systems. These sources are having random fluctuation of photons. Result gathered image has spatial and temporal randomness. This noise is also called as quantum (photon) noise or shot noise. This noise obeys the Poisson distribution and is given as:

$$f(x,\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 \;, & x < 0 \end{cases}$$

Poisson distribution is a discrete probability distribution. Poisson distribution models the probability of the number of events occurring over a fixed time step given a known mean. The events are independent of the last time they occurred. On the x-axis are the discrete values for the events 0, 1, 2, 3, 4, and so on (often representing the number of times the event occurs), and on the y-axis is the probability that the phenomenon occurs that many times given a known mean.

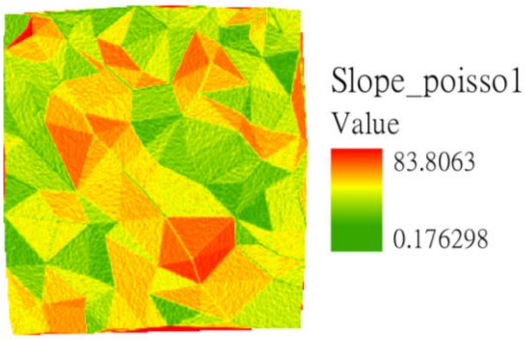


Figure 5.7 The slope map of sample 1 in Poisson distribution

5.2.3 Normal distribution

The noise follows the rule of normal distribution is also called as gaussian noise (electronic noise) because it arises in amplifiers or detectors. Gaussian noise caused by natural sources such as thermal vibration of atoms and discrete nature of radiation of warm objects. Gaussian noise generally disturbs the gray values in digital images.

Normal distribution models continuous random variables that commonly occur. Normal distribution is widely used and is applicable in many applications. It is built on the central limit theorem, which is based on the principle that the sum of the random variables is normally distributed if there are a large number of observations. The formula for normal distribution is as follows:

$$P(g) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{-\frac{(g-\mu)^2}{2\sigma^2}}$$

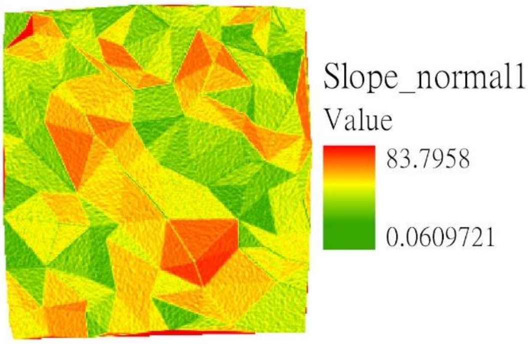


Figure 5.8 The slope map of sample 1 in normal distribution

5.2.4 Exponential distribution

Exponential noise distribution depends on the camera sensor. It is generally seen in the laser-based images. Exponential distribution is a continuous probability distribution. This noise obeys the exponential distribution and is given as:

$$f(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

From figure 5.9, as the mean increases, there is an exponentially greater chance of cell of DEM is replaced by distributed random variables so the noise of the noise map is also increased.

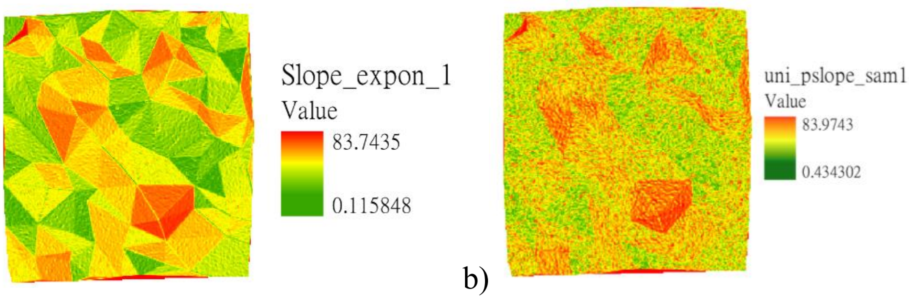


Figure 5.9 The slope map of sample 1 in exponential distribution,
a) mean: 1, b) mean: 5

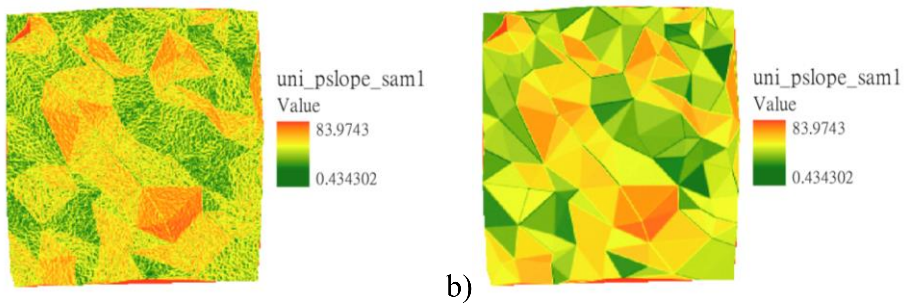
5.2.5 Uniform distribution

Uniform (Quantization) noise results when a continuous random variable is converted to a discrete one or when a discrete random variable is converted to one with fewer levels. In images, quantization noise often occurs in the acquisition process. Quantization noise appearance is inherent in amplitude quantization process. It is generally presenting due to analog data converted into digital data.

Uniform distribution is a continuous probability distribution in which all values within a specified interval have the same probability. The formula for uniform distribution is as follows:

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & for\ (a \le k \le b) \\ 0,\ for\ (x < a) or\ (x > b) \end{cases}$$

As can been seen from figure 5.10, as the range of random value increase, so does the noise of the slope map.



a) b)

Figure 5.10 The slope map of sample 1 in uniform noise,
a) Maximum: 10, Minimum: 0, b) Maximum: 1, Minimum: 0

5.3 The effect of colour schemes on indicating the gradient of extracted slope values

As mentioned above, slope has both magnitude and direction. That is, slope is a vector consisting of gradient and aspect. The term aspect is defined as the direction of the biggest slope vector on the tangent plane projected onto the horizontal plane. Aspect is the bearing (or azimuth) of the slope direction, and its angle ranges from 0 to 360◦. In this context, it is referred to an aspect map in ArcGIS Pro. The term slope is used to refer to the gradient, which mean the slope map generated in ArcGIS Pro only consider the gradient of extracted slope. With respect to the definition of the gradient, the lower the slope value, the flatter the terrain; the higher the slope value, the steeper the terrain. Therefore, the selectin of colour scheme should be considering the logical trend of slope magnitude.

In contrast to aspect map, the colour scheme of aspect map should allow the reader to easily distinguish different directions by the illogical ordered hues shown in Figure 5.11, whereas the slope map cannot. If the slope map is in illogically ordered hues, the reader have to spend effort to discriminate the relationship of the values and cannot represent the trend of slope value. In this case, sequential colour scheme would be ideal for the demonstration of the gradient shown as figure 5.12.

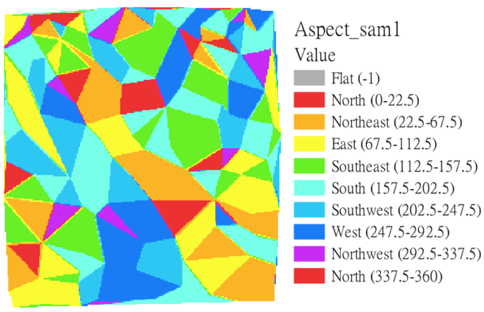
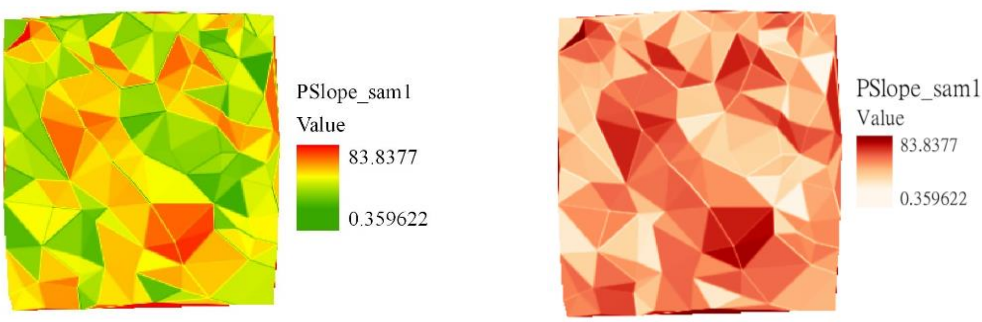


Figure 5.11 An aspect map of sample 1 in discrete colour scheme



a) Slope colour scheme b) Orange-Red (continuous)

Figure 5.12 The slope map in sequential colour scheme

CHAPTER 6
DISCUSSION

6.1 Detection of gross error in raster-based DEM based on slope information

Slope is the fundamental attribute of a terrain point and, therefore, slope information can probably serve as the basis for the development of suitable algorithms for gross error detection. The computation of the slope of each grid point in different directions does not present a real problem. In this view, it appears promising to make use of slope information as the basis for developing algorithms for detection of gross errors.

Hannah (1981) developed an algorithm for the reduction of gross errors, based on the absolute slope values. The principle of Hannah's algorithm can be described briefly as follows. As a first step, the slopes between the point under investigation, say P, and its neighbours (eight if not located on the boundary) are computed. Once this has been done for the whole data set, three tests are carried out on the slopes.

1. Slope constraining test: To checks the (eight) slopes immediately surrounding P to see if they are reasonable, that is, whether they exceed the predefined threshold value or not.

2. Local neighbour slope consistency test: To checks the four pairs of slopes crossing P to see if the absolute value of the difference in slope in each pair exceeds the given threshold value.

3. Distant neighbour slope consistency test: It is similar to the second test. This test checks whether pairs of slopes approaching a point across each of the eight neighbours are consistent.

The results of these three tests are used as the basis to judge whether a point is accepted or rejected based on the absolute slope values. Obviously, absolute slope values and slope differences will vary from place to place. For example, in an area with rough terrain, absolute slope differences will be larger than those found in smooth areas. Absolute values of slopes in steep areas will be larger than those found in flat areas. That is, it is not feasible for an absolute threshold value to be suitable overall for an area of interest except in a homogenous area. For this reason, Li (1990) tried to make relative thresholds for his algorithm development and his algorithm will be presented in the following.

6.1.1 The principle of gross error detection based on an adaptive threshold

The algorithm developed by Li (1990) is based on the concept of slope consistency. Instead of absolute values of slope and slope changes, relative values are considered. Furthermore, a statistic is taken from these relative values and is then used as the threshold value to measure the validity of a data point instead of using a predefined value. Thus, this algorithm is adaptive to any data set.

As shown in Figure 6.1, data point P can be defined by its rowand column number, (I , J) within the height matrix. Its eight immediate neighbors — points 5, 6, 7, 10, 12, 15, 16, and 17 — can also be defined by row and column as (I + 1, J − 1), (I + 1, J), (I + 1, J + 1), (I , J − 1), (I , J + 1), (I − 1, J − 1), (I − 1, J), and (I −1, J +1). From these eight points and point P itself, six slopes can be computed in both the row (i.e., I) and the column (i.e., J) directions. Taking the row direction as an example, six slopes — those between points 5 and 6, 6 and 7, 10 and P, P and 12, 15 and 16, as well as 16 and 17 — can be computed. From the set of six slope values, three slope changes can then be computed. For example, the slope changes at points 6, P, and 16 can be computed from those values mentioned above. These initial values are given in an absolute sense and will vary from place to place. Therefore, some relative values need to be computed from them.

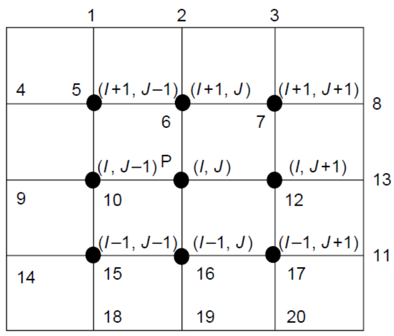


Figure 6.1 Point P in the original grid data and its neighbours (Li, et al., 2005)

If there is no gross error at point P, then for the same direction (e.g., the row direction), the difference in the slope change (DSC) at point P and that at its immediate neighbour (e.g., point 6 or 16) located in the row direction will be consistent, even though the absolute values of slope and slope change may vary from place to place. Therefore, these differences in slope change are the relative values that are being searched for and can be used as the basis of a method for detecting gross errors.

That is, for each point except those along the boundary, two DSC values can be computed from the three slope changes in each direction. The DSC values from all the data points will be used as the basic information for this algorithm. From these DSC values, a statistic will be computed, and it will then be used to construct the required threshold value. Then, this threshold value will be used as the basis on which a judgment is made as to whether or not a point has a gross error in elevation. For example, if all four DSC values centred at P exceed the threshold value, then P will be suspected of containing gross errors.

6.1.2 Computation of an adaptive threshold

In this computation, first of all, the DSC values in both row and column directions are computed and then these DSC values are used to compute an adaptive threshold. The computation of the slope (Slope) in the J direction, for example, is as follows:

$$Slope_j(I+1,J-1) = \frac{Z(I+1,J) - Z(I-1,J-1)}{Dist(J-1,J)}$$

where Dist(J − 1, J) is the distance between the nodes at (I + 1, J) and (I + 1, J − 1), that is, equal to the grid interval.

Similarly the values $Slope_j$ (I + 1, J), $Slope_j$ (I , J − 1), $Slope_j$ (I , J), $Slope_j$ (I − 1, J − 1), and $Slope_j$ (I − 1, J) can be calculated. The computation of slopes in the other (I) direction is similar. After calculating the slopes, three slope changes (SlopeC) in each direction can be computed. For example, in the J direction, the computation is as follows:

$$SlopeC_j(I,J) = Slope_j(I,J) - Slope_j(I,J-1)$$

Also, $SlopeC_j(I+1,J)$ and $SlopeC_j(I-1,J)$ can be computed similarly. The computation of slope changes in the I direction is also similar. After this, two differences in slope change (DSlopeC) for the point (I, J) in each direction can be computed as follows:

J direction:

$$DSlopeC_j\,(I\,,J\,,1) \;=\; SlopeC_j\,(I\,,J) \;-\; SlopeC_j\,(I\;+\;1,J)$$

$$DSlopeC_j\,(I\,,J\,,2) \;=\; SlopeC_j\,(I\,,J) \;-\; SlopeC_j\,(I-\;1,J)$$

I direction:

$$DSlopeC_j\,(I\,,J\,,1)\;=\;SlopeC_j\,(I\,,J)\;-\;SlopeC_j\,(I\,,J+\;1)$$

$$DSlopeC_j\,(I\,,J\,,2)\;=\;SlopeC_j\,(I\,,J)\;-\;SlopeC_j\,(I,J-\;1)$$

All DSC values calculated from all the data points will be used for computation of a statistic, which will then be used as threshold for acceptance or rejection of the point. For such a statistic, the absolute mean, the range (biggest minus smallest), the mode, the RMSE, as well as the standard deviation and mathematical mean are all possible options. Li (1988) made a thorough analysis on the possible statistics and Li (1990) made some observations from experiments and then concluded that RMSE is as good as the combination of the mathematical mean and standard deviation. Thus, the threshold value is simply K times RMSE of the DSC values, that is,

$$DSC_T\;=\;K\;\times\;RMSE(DSC)$$

where K is a constant. It has been found that the DSC values are quite normally distributed and thus a value of 3 has been used for the constant K. There are three possible ways to compute RMSE values:

1. Compute the only RMSE value from all DSC values at all the data points in all directions.
2. Compute four RMSE values from the DSC values at all the data points, one for each of the four directions (above, left, below, and right) defining each data point.
3. Compute two RMSE values, one of which is related to the row (i.e., the I) direction and the other to the column (i.e., the J) direction. In this case, the two DSC values of each point in the same direction, say the J direction, are added together to get a new value and the RMSE value can be computed from these new values.

All the methodology described above is designed to judge whether or not a point has a gross error. A particular threshold value for an individual direction is used as the basis for judgment. If the threshold is exceeded, then this point is regarded as unnatural in the neighbourhood and is suspected of having a gross error.

CHAPTER 7
CONCLUSION

This report discussed the effect of random errors on slope values by performing slope calculation to generate slope map based on DEM and adding random noise manually. By comparing the resulting maps in the different methods of calculating slope, it is clear that the geodesic method is an ideal method for small scale DEM requiring consideration of the curvature of the Earth. For large scale DEM, both geodesic and planner method can also be adopted. Also, the cause and the effect of errors of DEM on calculating slope by creating a raster dataset of random values with different distribution have been discussed. For the colour schemes on indicating the gradient of extracted slope values, the sequential colour scheme would be ideal for the demonstration of the gradient, while the discrete colour scheme is better to demonstrate the direction alone because it is easy to distinguish the different. In addition, the detection methods of gross error in raster-based DEM using slope information were discussed.

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